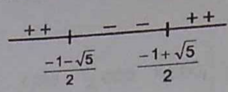
$x \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$ ....(iii)



from equation (i), (ii) and (iii)

solution of inequation is $\left(\frac{-1+\sqrt{5}}{2}, 1\right)$

7.25 (2)

$f(x) = 3|\sin x| - 4\,|\cos x|$

Since f(x) is periodic with period π. So for range analysis we can analyze in $[0, \pi]$

$$f(x) = \begin{cases} 3\sin x - 4\cos x \;, & x \in \left[0, \frac{\pi}{2}\right) \\ 3\sin x + 4\cos x \;, & \left[\frac{\pi}{2}, \pi\right] \end{cases}$$

In $\left[0, \frac{\pi}{2}\right)$, sinx, and – cos x is increasing function, hence 3 sin x – 4 cos x will be increasing in $\left[0, \frac{\pi}{2}\right)$

In $\left[\frac{\pi}{2}, \pi\right]$, sin x and cos x is decreasing, hence 3 sin x + 4 cos x will be decreasing

Hence for range we will check f(0), $f\left(\frac{\pi}{2}\right)$ and $f(\pi)$

$\therefore$ $f(0) = -4.$

$f\left(\frac{\pi}{2}\right) = 3$

$f(\pi) = -4$

Hence range of $f(x) = [-4, 3]$

7.26 (2)

$$f(x) = [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] - 3x + \sin 3\pi x - 1$$

$$= x - \{x\} + x + \frac{1}{3} - \left\{x + \frac{1}{3}\right\} + x + \frac{2}{3} - \left\{x + \frac{2}{3}\right\} - 3x + \sin 3\pi x - 1$$

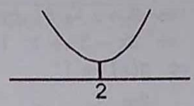
$$= \sin 3\pi x - \left(\{x\} + \left\{x + \frac{1}{3}\right\} + \left\{x + \frac{2}{3}\right\}\right)$$

$\sin 3\pi x$ is periodic with period $\frac{2}{3}$

$\{x\} + \left\{x + \frac{1}{3}\right\} + \left\{x + \frac{2}{3}\right\}$ is periodic with period $\frac{1}{3}$

$\therefore$ Period of $f(x) = \text{LCM}\left(\frac{2}{3}, \frac{1}{3}\right) = \frac{2}{3}$

7.27 **(1)**
$f(x) = x^2 - 4x + 5$
f(x) is a quadratic polynomial
f(x) is decreasing in $(-\infty, 2]$ and increasing in $[2, \infty)$.
Range of f(x) is $[1, \infty)$
hence f(x) will be invertible for
$X = (-\infty, 2] \rightarrow Y = [1, \infty)$
$X = [2, \infty) \rightarrow Y = [1, \infty)$



for $f^{-1}(x)$
$f(f^{-1}(x)) = x$
$\Rightarrow (f^{-1}(x))^2 - 4 . f^{-1}(x) + 5 = x$
$(f^{-1}(x))^2 - 4 . f^{-1}(x) + 5 - x = 0$

$$f^{-1}(x) = \frac{4 \pm \sqrt{16 - 4(5-x)}}{2}$$

$$f^{-1}(x) = \frac{4 \pm \sqrt{4x - 4}}{2}$$

$$f^{-1}(x) = 2 \pm \sqrt{x-1}$$

$f : (-\infty, 2] \rightarrow [1, \infty)$, $f^{-1}(x) = 2 - \sqrt{x-1}$

$f : [2, \infty) \rightarrow [1, \infty)$, $f^{-1}(x) = 2 + \sqrt{x-1}$

7.28 **(1)**
$f'(x) = x^2 + x + a$
$f'(x) \geq 0$

$$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \left(a - \frac{1}{4}\right) \geq 0$$

Hence $a \geq \frac{1}{4}$

7.29 **(1)**
Obvious

7.30 **(1)**
Obvious

8. LIMIT OF FUNCTION

8.1 (2)

As $x \to 2^- \Rightarrow 3 - x \to 1^+$ i.e. $g(x) \to 1^+$

As $x \to 2^+ \Rightarrow 2x - 3 \to 1^+$ i.e. $g(x) \to 1^+$

$\therefore\ x \to 2 \Rightarrow g(x) \to 1^+$

$$\lim_{x\to 2} f(g(x)) = \lim_{g(x)\to 1^+} f(g(x))$$

$$= \lim_{x\to 1^+} f(x) \quad = \lim_{x\to 1^+} (3x - 1) = 2$$

8.2 (3)

since function is not define as $x \to 0^-$ because $[x + 1] = 0$, therefore limit does not exist

8.3 (2)

given limit

$$= \lim_{x\to\infty} \frac{\tan^{-1}\frac{1}{x}}{\sin^{-1}\frac{1}{x}},\ x > 0$$

$$= \lim_{t\to 0} \frac{\tan^{-1} t}{\sin^{-1} t} = \lim_{t\to 0} \frac{\frac{\tan^{-1} t}{t}}{\frac{\sin^{-1} t}{t}} = 1$$

8.4 (1)

$$\lim_{x\to 0^+} \frac{-1+\sqrt{1+4(\tan x - \sin x)}}{-1+\sqrt{1+4x^3}} \qquad \left(\frac{0}{0}\text{ form}\right)\left(\frac{0}{0}\text{ रूप}\right)$$

Rationalizing numerator and denominator

$$= \lim_{x\to 0^+} \frac{4(\tan x - \sin x)(1+\sqrt{1+4x^3})}{4x^3(1+\sqrt{1+4(\tan x - \sin x)})}$$

$$= \lim_{x\to 0^+} \left(\frac{\sin x(1-\cos x)}{x^3 \cos x}\right)\left(\frac{(1+\sqrt{1+4x^3})}{(1+\sqrt{1+4(\tan x - \sin x)})}\right)$$

$$= \lim_{x\to 0^+} \left(\frac{\sin x}{x}\right) \cdot \left(\frac{2\sin^2\frac{x}{2}}{4\times\frac{x^2}{4}}\right) \cdot \left(\frac{1}{\cos x}\right) \cdot \left(\frac{(1+\sqrt{1+4x^3})}{(1+\sqrt{1+4(\tan x - \sin x)})}\right)$$

$$= 1 . \frac{1}{2} . 1 \frac{(1+1)}{(1+1)} = \frac{1}{2}$$

8.5 **(3)**

$$\lim_{x\to 0} \frac{\ell n(1+2x-3x^2+4x^3)}{\ell n(1-5x+6x^2-7x^3)} \qquad \left(\frac{0}{0}\text{ form}\right)\left(\frac{0}{0}\text{ रूप}\right)$$

$$= \lim_{x\to 0}\left(\frac{\ell n(1+2x-3x^2+4x^3)}{(2x-3x^2+4x^3)}\right)\times\left(\frac{(-5x+6x^2-7x^3)}{\ell n(1-5x+6x^2-7x^3)}\right)\times\left(\frac{2x-3x^2+4x^3}{-5x+6x^2-7x^3}\right)$$

$$= 1 \times 1 \times \left(-\frac{2}{5}\right) = -\frac{2}{5}$$

8.6 **(2)**

$$\lim_{x\to 0} \frac{3^x-1}{\sqrt{1+x}-1} \qquad \left(\frac{0}{0}\text{ form}\right)\left(\frac{0}{0}\text{ रूप}\right)$$

By rationalization

$$= \lim_{x\to 0} \frac{3^x-1}{\sqrt{1+x}-1}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x\to 0}\left(\frac{3^x-1}{x}\right)\{\sqrt{1+x}+1\}$$

$= 2\,\ell n\,3$

8.7 **(4)**

$$\lim_{x\to -n}\frac{x^7+n^7}{x+n} = 7 \qquad \left(\frac{0}{0}\text{ form}\right)\left(\frac{0}{0}\text{ रूप}\right)$$

$$\lim_{x\to -n}\frac{x^7-(-n)^7}{x-(-n)} = 7$$

$7(-n)^{7-1} = 7$

$7n^6 = 7$

$n^6 = 1$

$n = \pm 1$

8.8 **(3)**

$$\lim_{x\to 2}\frac{\tan\left\{\left(\frac{e^{x-2}-1}{x-2}\right)(x-2)\right\}}{\left(\frac{\ell n(1+x-2)}{x-2}\right)(x-2)} = \lim_{x\to 2}\frac{\tan(x-2)}{(x-2)} = 1$$

8.9 **(4)**

$$\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{ex}{2}}{11x^2} \qquad \left(\frac{0}{0}\text{ form}\right)\left(\frac{0}{0}\text{ रूप}\right)$$

By expansion

$$= \lim_{x\to 0}\frac{e\left(1-\frac{x}{2}+\frac{11}{24}x^2+.....\right)-e+\frac{ex}{2}}{11x^2} = \frac{e}{24}$$

8.10 (2)

$$\lim_{x\to 0^-} \frac{2\left[1-\left(1-\frac{7x}{256}\right)^{1/8}\right]}{2\left[\left(1+\frac{5x}{32}\right)^{1/5}-1\right]} = \lim_{x\to 0^-} \frac{1-\left(1-\frac{1}{8}\cdot\frac{7x}{256}+.....\right)}{\left(1+\frac{1}{5}\cdot\frac{5x}{32}+.......\right)-1} = \frac{\frac{1}{8}\cdot\frac{7}{256}}{\frac{1}{5}\cdot\frac{5}{32}} = \frac{7}{64}$$

8.11 (4)

Let $y = \lim_{x\to 0^+} (\sin x)^{1/\ell nx}$

$$\ell ny = \lim_{x\to 0^+} \frac{\ell n \sin x}{\ell nx} \qquad \left(\frac{\infty}{\infty} \text{ form रूप}\right)$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\sin x}\cos x}{1/x}$$

$$= \lim_{x\to 0^+} \frac{x}{\tan x}$$

$$\ell ny = \lim_{x\to 0^+} \frac{1}{\sec^2 x} = 1$$

$y = e$

8.12 (1)

$$\lim_{x\to\infty} \frac{(\ell nx)^2}{x^2} \qquad \left(\frac{\infty}{\infty} \text{ form}\right)\left(\frac{\infty}{\infty} \text{ रूप}\right)$$

$$= \lim_{x\to\infty} \frac{2\ell nx.\left(\frac{1}{x}\right)}{2x}$$

$$= \lim_{x\to\infty} \frac{\ell nx}{x^2} \qquad \left(\frac{\infty}{\infty} \text{ form}\right)\left(\frac{\infty}{\infty} \text{ रूप}\right)$$

$$= \lim_{x\to\infty} \frac{\left(\frac{1}{x}\right)}{2x}$$

$$= \lim_{x\to\infty} \frac{1}{2x^2} = 0$$

8.13 (2)

$$\lim_{x\to\infty} \frac{2x-3}{x} = \lim_{x\to\infty}\left(2-\frac{3}{x}\right) = 2$$

and $\lim_{x\to\infty} \frac{2x^2+5x}{x^2} = \lim_{x\to\infty}\left(2+\frac{5}{x}\right) = 2$

$\therefore$ Using sandwitch theorem, $\lim_{x\to\infty} f(x) = 2$

8.14 (4)

$$\lim_{x\to 0}\frac{\sqrt[3]{1+\tan x}-\sqrt[3]{1-\tan x}}{x} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 0}\frac{\frac{1}{3}(1+\tan x)^{-\frac{2}{3}}.\sec^2 x+\frac{1}{3}(1-\tan x)^{\frac{-2}{3}}.\sec^2 x}{1} = \frac{1}{3}(1)^{-\frac{2}{3}}.1+\frac{1}{3}(1)^{-\frac{2}{3}}.1 = \frac{2}{3}$$

8.15 (1)

$$\lim_{x\to\frac{\pi}{2}} \tan x\,.\,\ell n \sin x = \lim_{x\to\frac{\pi}{2}}\frac{\ell n \sin x}{\cot x} \quad \left(\frac{0}{0}\text{ form}\right)$$

By L-Hospital rule

$$= \lim_{x\to\frac{\pi}{2}}\frac{\frac{1}{\sin x}\cos x}{-\operatorname{cosec}^2 x} = \lim_{x\to\frac{\pi}{2}} -\sin x\cos x = 0$$

8.16 (2)

$$\lim_{n\to\infty}\left(2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots+\frac{1}{2^n}}\right) = \lim_{n\to\infty} 2^{\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}}} = 2^1$$

8.17 (1)

When y = x – 1, we get x = y + 1

As (x, y) → (1, 0) along the line x = y + 1

$$\text{so } \lim_{y\to 0}\frac{y^3}{(y+1)^3-y^2-1} = \lim_{y\to 0}\frac{y^3}{y^3+2y^2+3y} = \lim_{y\to 0}\frac{y^2}{y^2+2y+3} = \frac{0}{3} = 0$$

8.18 (1)

$$\lim_{x\to 0}\frac{x\left[1+a\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\right)\right]-b\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}\right)}{x^3}$$

$$\Rightarrow \lim_{x\to 0}\frac{\left(x+ax-\frac{ax^3}{2!}+\frac{ax^5}{4!}-\ldots\ldots\right)-bx+\frac{bx^3}{3!}-\frac{bx^5}{5!}}{x^3}$$

$$\Rightarrow \lim_{x\to 0}\frac{x(1+a-b)+x^3\left(-\frac{a}{2!}+\frac{b}{3!}\right)+x^4(\ldots\ldots)}{x^3} = 1$$

by comparing the two sides

$= 1 + a - b = 0 \qquad \Rightarrow a - b = -1$

$$-\frac{a}{2!}+\frac{b}{3!} = 1$$

$b - 3a = 6$

from (1) and (2) $a = -\frac{5}{2}$ and $b = -\frac{3}{2}$

8.19 **(2)**

$$\lim_{n\to\infty}\left(1+\sin\frac{a}{n}\right)^n \qquad (1^\infty \text{ form})$$

$$= e^{\lim_{n\to\infty} n\sin\frac{a}{n}}$$

$$= e^{\lim_{n\to\infty}\frac{\sin\frac{a}{n}}{1/n}}$$

$$= e^{\lim_{n\to\infty}\frac{\cos\frac{a}{n}\cdot\left(-\frac{a}{n^2}\right)}{\left(-\frac{1}{n^2}\right)}} = e^a$$

8.20 **(1)**

$$\lim_{x\to1}\left(\frac{(x-1)(x^{n-1}+x^{n-2}+......+1)}{n(x-1)}\right)^{\frac{1}{x-1}} \qquad (1^\infty \text{ form})$$

$$= e^{\lim_{x\to1}\left(\frac{x^n-1}{n(x-1)}-1\right)^{\frac{1}{x-1}}}$$

$$= e^{\lim_{x\to1}\left(\frac{x^{n-1}+x^{n-2}+........+1-n}{n(x-1)}\right)}$$

$$= e^{\lim_{x\to1}\left(\frac{x^{n-1}-1}{x-1}+\frac{x^{n-2}-1}{x-1}+.....+\frac{x-1}{x-1}+\frac{1-1}{x-1}\right)\frac{1}{n}}$$

$$= e^{((n-1)+(n-2)+(n-3)+....+2+1)\frac{1}{n}} \qquad = e^{\frac{n-1}{2}}$$

8.21 **(2)**

$$\lim_{x\to\infty}\left\{\sqrt{x^4-x^2+1}-ax^2-a\right\}$$

By rationalization

$$\lim_{x\to\infty}\left\{\sqrt{x^4-x^2+1}-(ax^2+a)\right\}\times\frac{\left\{\sqrt{x^4-x^2+1}+(ax^2+a)\right\}}{\left\{\sqrt{x^4-x^2+1}+(ax^2+a)\right\}}$$

$$\lim_{x\to\infty}\frac{x^4-x^2+1-a^2(x^4+1+2x^2)}{\sqrt{x^4-x^2+1}+(ax^2+a)} = \text{finite value}$$

$$\lim_{x\to\infty}\frac{x^4(1-a^2)-x^2(1+2a^2)+(1-a^2)}{x^2\left\{\sqrt{1-\frac{1}{x^2}+\frac{1}{x^4}}+a+\frac{a}{x^2}\right\}} = \text{finite value}$$

$\because\ 1-a^2=0 \Rightarrow a=\pm1,$

$$\frac{-(1+2a^2)}{1+a} = \text{finite value}$$

$\therefore\ a\neq-1,\ a=1$

8.22 (1)

$$\lim_{x\to 0}\frac{e^x - e^{x\sec x}}{x+\tan x} \qquad \left(\frac{0}{0}\text{ form}\right)\left(\frac{0}{0}\text{ रूप}\right)$$

$$= \lim_{x\to 0}\frac{(e^x-1)-(e^{x\sec x}-1)}{(x+\tan x)}$$

$$= \lim_{x\to 0}\left(\frac{e^x-1}{x+\tan x}\right)-\left(\frac{e^{x\sec x}-1}{x+\tan x}\right)$$

$$= \lim_{x\to 0}\left(\frac{e^x-1}{x}.\frac{x}{x+\tan x}\right)-\left(\frac{e^{x\sec x}-1}{x\sec x}.\frac{x\sec x}{x+\tan x}\right)$$

$$= \lim_{x\to 0}\left(\frac{e^x-1}{x}.\frac{1}{1+\tan x/x}\right)-\left(\frac{e^{x\sec x}-1}{x\sec x}.\frac{\sec x}{1+\tan x/x}\right)$$

$$= \left(1.\frac{1}{1+1}\right)-\left(1.\frac{1}{1+1}\right) = 0$$

8.23 (2)

Put x = 1 + h

$$= \lim_{h\to 0}\frac{(\log(1+1+h)-\log 2)(3.4^h-3(1+h))}{[(7+1+h)^{1/3}-(1+3(1+h))^{1/2}]\sin\pi(1+h)}$$

$$= \lim_{h\to 0}\frac{[\log(2+h)-\log 2]\,[3(4^h-1)-3h]}{[(8+h)^{1/3}-(4+3h)^{1/2}]\,(-\sin\pi h)}$$

$$= -\lim_{h\to 0}\frac{\log\left(1+\frac{h}{2}\right)(3.(4^h-1)-3h)}{2\left\{\left(1+\frac{h}{8}\right)^{1/3}-\left(1+\frac{3h}{4}\right)^{1/2}\right\}\sin\pi h}$$

$$= -\lim_{h\to 0}\frac{\frac{1}{\pi}.\frac{\log\left(1+\frac{h}{2}\right)}{\frac{h}{2}}\left\{\frac{3(4^h-1)}{h}-3\right\}}{\left(\frac{1}{24}-\frac{3}{8}\right)}$$

$$= -\frac{\frac{1}{\pi}.3(\log 4-1)}{\frac{1-9}{24}}$$

$$= \frac{9}{\pi}\log\frac{4}{e}$$

8.24 **(1)**

$$\lim_{x\to 0}\left(\frac{e^{x\ell n(3^x-1)}-(3^x-1)^x\sin x}{e^{x\ell n x}}\right)^{1/x}$$

$$=\lim_{x\to 0}\left(\frac{(3^x-1)^x-(3^x-1)^x\sin x}{x^x}\right)^{1/x}=\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)(1-\sin x)^{1/x}$$

$$=\left(\lim_{x\to 0}\frac{3^x-1}{x}\right)\lim_{x\to 0}(1-\sin x)^{\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{x}\right)}=\ell n\,3.\,e^{-1}=\frac{\ell n 3}{e}$$

8.25 **(2)**

Let $y=\lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{1/x}$

$=\lim_{x\to\infty}(\cot^{-1}x)^{1/x}$ $\quad(0^0$ form$)$

$\ell n y=\lim_{x\to\infty}\frac{\ell n\cot^{-1}x}{x}$ $\quad\left(\frac{\infty}{\infty}\text{ form}\right)$

$=\lim_{x\to\infty}-\frac{1}{(1+x^2)\cot^{-1}x}$

$=-\lim_{x\to\infty}\frac{(1+x^2)^{-1}}{\cot^{-1}x}$ $\quad\left(\frac{0}{0}\text{ form}\right)$

$$=-\lim_{x\to\infty}\frac{\frac{-2x}{(1+x^2)^2}}{-\frac{1}{1+x^2}}$$

$=-2\lim_{x\to\infty}\frac{x}{1+x^2}$

$=-2\lim_{x\to\infty}\frac{1}{2x}=0$

$y=e^0=1$

8.26 **(1)**

$\lim_{x\to a}\frac{\log_e(1+6(f(x))}{3f(x)}$ $\quad\left(\frac{0}{0}\text{ form}\right)$ $\quad\left(\frac{0}{0}\text{ रूप}\right)$

By L-Hospital rule

$$\lim_{x\to a}\frac{\frac{6f'(x)}{1+6f(x)}}{3f'(x)}=\lim_{x\to a}\frac{2}{1+6f(x)}=2$$

8.27 **(4)**

The Statement-1 is

finite no. between

Thus $\lim_{x\to 0}x\sin\frac{1}{x}$

The Statement-2 i

limit $\lim_{x\to 0}\frac{\sin x}{x}=$

8.28 **(1)**

$\lim_{x\to 0}\left(\frac{g(2-x^2)}{g(2)}\right.$

$=e^{\lim_{x\to 0}\left(\frac{g(2-x^2)}{g(2)}\right.}$

$=e^{\lim_{x\to 0}\left(\frac{g(2-x^2)-}{x^2}\right.}$

$=e^{-g'(2)\times\frac{4}{g(2)}}$

Resonance Educating

8.27 **(4)**

The Statement-1 is false since as $x \to 0$. The function $x \sin \frac{1}{x}$ = (a quantity approaching zero) × (any finite no. between 0 to 1)

Thus $\lim_{x\to 0} x \sin \frac{1}{x} \to 0$

The Statement-2 is true since it is equivalent to standard

limit $\lim_{x\to 0} \frac{\sin x}{x} = 1$

8.28 **(1)**

$$\lim_{x\to 0}\left(\frac{g(2-x^2)}{g(2)}\right)^{\frac{4}{x^2}} \qquad (1^\infty \text{ form})$$

$$= e^{\lim_{x\to 0}\left(\frac{g(2-x^2)}{g(2)}-1\right)\frac{4}{x^2}}$$

$$= e^{\lim_{x\to 0}\left(\frac{g(2-x^2)-g(2)}{x^2}\right)\frac{4}{g(2)}}$$

$$= e^{-g'(2)\times\frac{4}{g(2)}} = e^{5\times\frac{4}{-40}} = e^{-\frac{1}{2}}$$

9. CONTINUITY & DERIVABILITY

9.1 (4)

$a\,|\sin^7 x|$ is differentiable at $x = 0$ and d.c. is 0 for all $a \in R$ and $c\,|x|^5$ is differential at $x = 0$ and its d.c. is 0 for all $x \in R$ but at $x = 0$ L.H. derivative of $be^{|x|} = -b$ R.H. dirivatives $= b$

$\therefore$ for $be^{|x|}$ to be differentiable at $x = 0$

$b = -b \Rightarrow b = 0$

9.2 (1)

$g(x)$ is discontinuous at $x = -3$

$f(x)$ is discontinuous at $x = 1$

But $g(x) \neq 1$. Hence their are only one point of discontinuity

9.3 (2)

$f(1) = a + b$

$$f(1 + h) = \frac{|1+h-1|}{1-(1+h)} + a = -1 + a$$

$\because$ Function is continuous

$\therefore$ $f(1) = f(1 + h)$

$= a + b = -1 + a \Rightarrow b = -1$

$$\text{Now } f(1 - h) = \frac{|1-h-1|}{1-(1-h)} + b = \frac{h}{h} + b = 1 + b$$

$\therefore$ $a + b = 1 + b \Rightarrow a = 1$

9.4 (1)

$\because$ $f(x)$ is continuous in $[0, \pi]$

so it is also continuous at $x = \pi/4$ and $x = \pi/2$

$$\therefore \lim_{x\to\pi/4^-} f(x) = \lim_{x\to\pi/4^+} f(x)$$

$$\Rightarrow \pi/4 + a = \pi/2 + b \qquad \ldots(1)$$

$$\text{and } \lim_{x\to\pi/2^-} f(x) = \lim_{x\to\pi/2^+} .f(x)$$

$$\Rightarrow 0 + b = -a - b \qquad \ldots(2)$$

By solving (1) and (2)

$\Rightarrow a = \pi/6,\ b = -\pi/12$

9.5 (4)

We have $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} \sin(\log_e|-h|) = \lim\limits_{h\to 0} \sin(\log_e h)$ which does not exist but lies between -1 and 1. Similarly $\lim\limits_{x\to 0^+} f(x)$ lies between -1 and 1 but cannot be determined.

at x = 0 and its d.c. is

veen

9.6 (1)

Since $f(1-0) = \lim_{x\to1} 3^x = 3$

$f(1+0) = \lim_{x\to1} (4-x) = 3$

and $f(1) = 3^1 = 3$

$\therefore\ f(1-0) = f(1+0) = f(1)$

$\Rightarrow f(x)$ is continuous at x = 1

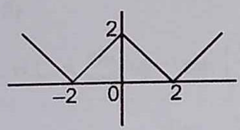
Again $f'(1+0) = \lim_{x\to1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to1} \frac{3^x-3}{x-1} = \lim_{h\to0} \frac{3^{1+h}-3}{h} = 3\lim_{h\to0} \frac{3^h-1}{h} = 3\log 3$

and $f'(1+0) = \lim_{x\to1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to1} \frac{4-x-3}{x-1} = -1$

$\therefore\ f'(1+0) \neq f'(1-0)$

$\Rightarrow f(x)$ is not differentiable at x = 1

9.7 (3)



9.8 (1)

When x < 0, $f(x) = \frac{x}{1-x}$

$\Rightarrow f'(x) = \frac{1}{(1-x)^2}$

which exist finitely for all x < 0

when x > 0, $f(x) = \frac{x}{1+x}$

$\Rightarrow f'(x) = \frac{1}{(1+x)^2}$

which exist finitely for all x > 0

from (1) and (2)

$\begin{cases} f'(0-0) = 1 \\ f'(0+0) = 1 \end{cases} \Rightarrow f'(0) = 1$

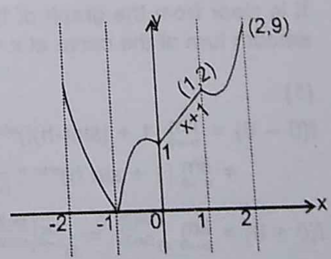
so f(x) is differentiable $\forall\ x \in R$.

9.9 (3)

Here,

$f(x) = |x+1|\{|x| + |x-1|\}$

$$f(x) = \begin{cases} (x+1)(2x-1) & ; \quad -2 \le x \le -1 \\ -(x+1)(2x-1) & ; \quad -1 \le x < 0 \\ (x+1) & ; \quad 0 \le x < 1 \\ (x+1)(2x-1) & ; \quad 1 \le x \le 2 \end{cases}$$



Thus the graph of f(x) is;

which is clearly, continuous for $x \in R$ and, differentiablity for $x \in R - \{-1, 0, 1\}$

9.10 (4)

(3) $f(x) = \begin{cases} 1 & , \ 0 \le x \le \frac{3\pi}{4} \\ \frac{2}{\sqrt{3}} \sin\left(\frac{4x}{9}\right), & \frac{3\pi}{4} < x < \pi \end{cases}$

$f\left(\frac{3\pi}{4}\right) = 1$

$f\left(\frac{3\pi}{4} + h\right) = \lim\limits_{h \to 0} \frac{2}{\sqrt{3}} \sin \frac{4}{9}\left(\frac{3\pi}{4} + h\right) = \frac{2}{\sqrt{3}} \sin \frac{\pi}{3} = 1$

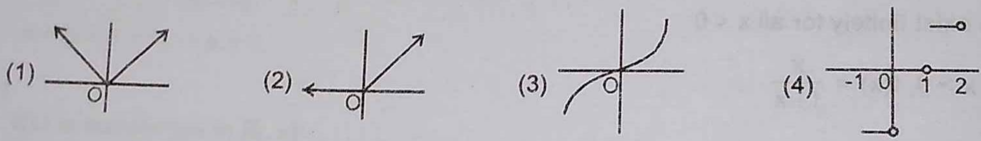
(4) $f(x) = \begin{cases} x \sin x & , \ 0 \le x \le \frac{\pi}{2} \\ \frac{\pi}{2} \sin(\pi + x), & \frac{\pi}{2} < x < \pi \end{cases}$

$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \sin \frac{\pi}{2} = \frac{\pi}{2}$

$f\left(\frac{\pi}{2} + h\right) = \lim\limits_{h \to 0} \frac{\pi}{2} \sin\left(\pi + \frac{\pi}{2} + h\right)$

$= \lim\limits_{h \to 0} \frac{\pi}{2}(-\cosh) = -\frac{\pi}{2}$

9.11 (4)



9.12 (2)

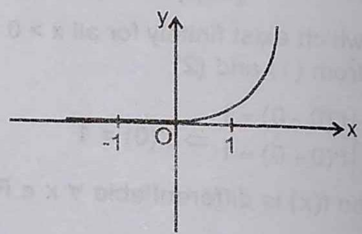
If $t \ge 0$ then $x = t$, $y = 2t^2$

$\therefore$ $y = 2x^2$, $x \ge 0$ $\quad (\because \ t = x)$

and for $t < 0$, $x = 3t$, $y = 0$, and $x < 0$ $(\because \ x = 3t)$

$\therefore$ The function is defined as

$f(x) = \begin{cases} 2x^2 & ; \ \text{if } 0 \le x \le 1 \\ 0 & ; \ \text{if } -1 \le x < 0 \end{cases}$



It is clear from the graph of f(x), f(x) is differentiable and continuous for all $x \in [-1, 1]$. Notice the smooth turn of the curve at x = 0

9.13 (1)

$f(0 - 0) = \lim\limits_{h \to 0} (1 + |\sin(-h)|)^{a/|\sin(-h)|}$

$= \lim\limits_{h \to 0} (1 + \sin h)^{a/\sin h} = e^a$

$f(0 + 0) = \lim\limits_{h \to 0} e^{\frac{\tan 2h}{\tan 3h}} = e^{\lim\limits_{h \to 0}\left(\frac{\tan 2h}{\tan 3h}\right)} = e^{\lim\limits_{h \to 0} \frac{2\sec^2 2h}{3\sec^2 3h}} = e^{2/3}$

now f(x) is continuous at x = 0

$\Rightarrow f(0 - 0) = f(0 + 0) = f(0) \Rightarrow e^a = e^{2/3} = b \Rightarrow a = 2/3, \ b = e^{2/3}$

9.14 (1)

$\lim_{x\to0} f(x) = f(0)$

$\lim_{x\to0} \frac{\cos(\sin x) - \cos x}{x^2} = a$

$\Rightarrow \frac{2}{x^2}\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x - \sin x}{2}\right) = a$

$\Rightarrow a = \lim_{x\to0} 2.\frac{\sin\left(\frac{\sin x + x}{2}\right)}{\frac{\sin x + x}{2}}.\frac{\sin\left(\frac{x - \sin x}{2}\right)}{\frac{x - \sin x}{2}}.\frac{1}{4}\left(\frac{\sin x + x}{x}\right)\left(\frac{x - \sin x}{x}\right)$

$= 2.1 . 1. \frac{1}{4}(1 + 1)(1 - 1) = 0$

9.15 (3)

L.H.L. = $\lim_{h\to0} \tan^{-1}\tan\left(\frac{\pi}{4} - h\right) = \frac{\pi}{4}$

R.H.L. = $\lim_{h\to0} \pi . \left[\frac{\pi}{4} + h\right] + 1 = 1$

Jump of discontinuity = $1 - \frac{\pi}{4}$

9.16 (2)

LHL (x = 0) = f(0) = RHL (x=0)

LHL = $\lim_{x\to0^-} \frac{\sqrt{1+px} - \sqrt{1-px}}{x} = \frac{2p}{2} = p$

$f(0) = -\frac{1}{2}$ = RHL

9.17 (2)

$f(x) = \begin{cases} \frac{x(3e^{1/x} + 4)}{2 - e^{1/x}}, & x \neq 0 \\ 0 & x = 0 \end{cases}$

$f'(0^-) = \lim_{h\to0} \frac{\frac{-h(3e^{-1/h} + 4)}{2 - e^{-1/h}} - 0}{-h} = 2$

$f'(0^+) = \lim_{h\to0} \frac{h\left(\frac{3e^{-1/h} + 4}{2 - e^{-1/h}}\right) - 0}{h} = -3$ not differentiable

$\lim_{x\to0} f(x) = 0 = \lim_{h\to0^+} f(x) = f(0)$ continuous

9.18 **(2)**

$$f(x) = \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}\cos x = \begin{cases} \frac{\pi}{2} + x \ , & x \in [-\pi, 0] \\ \frac{\pi}{2} - x \ , & x \in [0, \pi] \end{cases}$$

continous but not differentiable at x =0

9.19 **(1)**

$$f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}, \ f(0) = 0 \text{ and } f'(0) = 3$$

Put y = 0 and x = 3x, we get $f(x) = \frac{f(3x)+f(0)}{3} = \frac{f(3x)}{3}$

$$f'(x) = \lim_{h\to 0} = \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} = \frac{f\left(\frac{3x+3h}{3}\right)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{f(x)+f(3h)}{3} - \frac{f(3x)+f(0)}{3}}{h}$$

$$= \lim_{h\to 0} \frac{f(3h)-f(0)}{3h} = f'(0) = 3$$

$f(x) = 3x + c, \ f(0) = 0 \Rightarrow c = 0 \Rightarrow f(x) = 3x$

9.20 **(4)**

$$\because \quad \text{L.H.L.} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{e^{[x]+|x|} - 2}{[x]+|x|} = \frac{e^{-1}-2}{-1}$$

$$\text{and R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{e^{[x]+|x|} - 2}{[x]+|x|}$$

$$= \lim_{x\to 0^+} \frac{e^{x}-2}{x} \to -\infty \qquad \therefore \qquad \text{L.H.L} \neq \text{R.H.L}$$

$\therefore$ (4)

9.21 **(4)**

$$f(x) = \left|\left(x+\frac{1}{2}\right)[x]\right| = \begin{cases} -(2x+1) & , \ -2 \le x < -1 \\ |x+1/2| & , \ -1 \le x < 0 \\ 0 & , \ 0 \le x < 1 \\ x + 1/2 & , \ 1 \le x < 2 \\ 5 & , \ x = 2 \end{cases}$$

f(x) is discontinous at $x = -1, 0, \frac{1}{2}, 1, 2$

9.22 (3)

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}\left([x^2]+\sqrt{\{x\}^2}\right)$

$\Rightarrow \lim_{x\to 1^+}(1+0) = 1$

$\because \lim_{x\to 1^-} f(x) \Rightarrow \lim_{x\to 1^-}(0+1) = 1$

and $f(1) = 1 \Rightarrow \lim_{x\to 1} f(x) = f(1)$

$\therefore$ continuous at x = 1

similarly we check for another integers

9.23 (1)

Let $f(x) = \sqrt{\dfrac{4\cos^2 x}{2+\cos x}} \Rightarrow f(\pi) = 2$

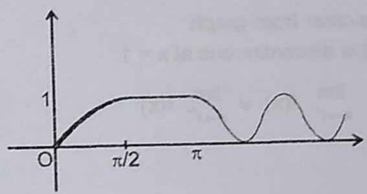
$\therefore \lim_{x\to\pi}\dfrac{1}{x-\pi}\left(\sqrt{\dfrac{4\cos^2 x}{2+\cos x}}-2\right) = \lim_{x\to\pi}\dfrac{f(x)-f(\pi)}{x-\pi} = f'(\pi)$

Now $f'(x) = \dfrac{4}{2\sqrt{\dfrac{4\cos^2 x}{2+\cos x}}}\;\dfrac{(2+\cos x)(-2\cos x\sin x)-\cos^2 x(-\sin x)}{(2+\cos x)^2}$

$f'(\pi) = 0$

9.24 (1)

$$g(x) = \begin{cases} f(x) & ; \; 0 \le x \le \dfrac{\pi}{2} \\ 1 & ; \; \dfrac{\pi}{2} < x \le \pi \\ \sin^2\dfrac{x}{2} & ; \; x > \pi \end{cases}$$



9.25 (2)

Given that f(x + y) = f(x).f(y) all $x \in R$

Putting x = y = 0 in (1), we get f(0) {f(0) – 1} = 0 ⇒ f(0) = 0 or f(0) = 1

If f(0) = 0, then f(x) = f(x + 0) = f(x).f(x) = 0 for all $x \in R$

Which is not true (given $f(x) \neq 0$)

So, f(0) = 1

$\therefore f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\dfrac{f(x)f(h)-f(x)}{h}$

$= f(x)\lim_{h\to 0}\dfrac{f(h)-1}{h} = f(x)\lim_{h\to 0}\dfrac{f(x)-f(0)}{h-0}$ $(\because f(0) = 1)$

$= f(x)f'(0) = 2f(x)$ $(\because f'(0) = 1)$

$\Rightarrow \dfrac{f'(x)}{f(x)} = 2$

Integrating both sides with respect to x and taking limit 0 to x

$\int_0^x \dfrac{f'(x)}{f(x)}dx = \int_0^x 2dx$

$\Rightarrow \ell n\, f(x) - \ell n\, f(0) = 2x \Rightarrow \ell n f(x) - \ell n 1 = 2x$

$\Rightarrow \ell n f(x) - 0 = 2x$

$\therefore f(x) = e^{2x}$

Clearly f(x) is everywhere continuous and differentiable

9.26 **(3)**

$g(t) = \lim_{x\to 0} (1 + a\tan x)^{t/x}$

$g(t) = e^{\lim_{x\to 0} \frac{t}{x} a \tan x} \quad = e^{\lim_{x\to 0} ta \frac{\tan x}{x}}$

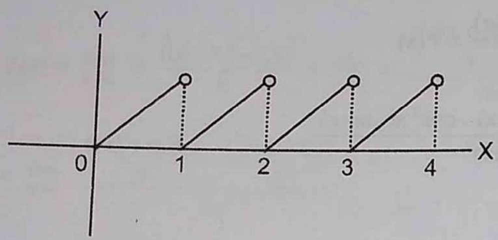
$g(t) = e^{ta} \quad = e^{ta}$

$g(x) = e^{ax}$

$\because a = 2,\ g(x) = e^{2x}$

$g(2) = e^{4}$

9.27 **(1)**



It is clear from graph

f(x) is discontinuous at x = 1

$\because \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

9.28 **(1)**

$f(x) = \left[\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right]$, which is discontinuous at points where $\sin\left(x + \frac{\pi}{4}\right) = 0, \pm\frac{1}{\sqrt{2}}, \pm 1$

9.29 **(2)**

The Statement -1 is true. Since $|f(x)| \le |x|$ for all x.

We have $|f(0)| \le 0$. But $|f(0)| \le 0$.

$\Rightarrow$ f(0) has to be zero

Now $|f(x)| \le |x|$

$\Rightarrow \lim_{x\to 0} |f(x)| \le \lim_{x\to 0} |x| = 0 \qquad \Rightarrow \lim_{x\to 0} |f(x)| \le 0$

But $\lim_{x\to 0} |f(x)|$ must be ≥ 0

$\Rightarrow \lim_{x\to 0} |f(x)|$ must be zero.

Thus, $\lim_{x\to 0} |f(x)| = |f(0)|$

$\Rightarrow |f(x)|$ is continuous at 0

Thus Statement-2 is also true

9.30 (3)

If f is differentiable at 'c' then f′(3) exists

$$\Rightarrow \lim_{h\to 0} \frac{f(c+h)-f(c)}{h} \text{ exists}$$

$$\Rightarrow \lim_{x\to 0} \frac{f(c)+f(h)-f(c)}{h} \text{ exists}$$

$$\Rightarrow \lim_{x\to 0} \frac{f(h)}{h} \text{ exists} \quad ...(1)$$

Now if 'd' is some other point then

$$f'(4) = \lim_{h\to 0} \frac{f(d+h)-f(d)}{h}$$

$$= \lim_{h\to 0} \frac{f(h)}{h} \text{ which exists from (1)}$$

and = f′(3)

Now any function is either differentiable nowhere or differentiable atleast at one point. From above if it is differentiable at one point, then it is differentiable for all x. Thus Statement-1 is true.

The Statement-2 is false since any function is either differentiable nowhere or is differentiable atleast at one point.

10. METHOD & DIFFERENTIATION

10.1 **(4)**

$$y = \tan^{-1}\left(\cot\frac{x}{2}\right)$$

$$= \frac{\pi}{2} - \cot^{-1}\cot\left(\frac{x}{2}\right)$$

$$= \frac{\pi}{2} - \frac{x}{2}$$

$$\frac{dy}{dx} = -\frac{1}{2}$$

10.2 **(2)**

$$f'(x) = \frac{\sqrt{2x+1}}{2x-1} + \frac{x}{\sqrt{2x+1}(2x-1)} - \frac{2x\sqrt{2x+1}}{(2x-1)^2}$$

$$f'(0) = -1$$

10.3 **(3)**

$$\frac{dx}{dt} = \frac{\cos t}{2+\cos t} + \frac{\sin^2 t}{(2+\cos t)^2}$$

at $t = \frac{\pi}{2}$, $\frac{dx}{dt} = \frac{1}{4}$

$$\frac{dy}{dt} = \frac{-2\sin t}{1+\cos t} + \frac{2\cos t\sin t}{(1+\cos t)^2}$$

at $t = \frac{\pi}{2}$, $\frac{dy}{dt} = -2$

so $\frac{dy}{dx} = -8$

10.4 **(1)**

$y\ell nx + x\ell ny = x^2$

$$\Rightarrow \quad \frac{y}{x} + y'\ell nx + \frac{x}{y}y' + \ell ny = 2x$$

$$\Rightarrow \quad y' = \frac{2x^2y - y^2 - xy\ell ny}{xy\ell nx + x^2}$$

$y'(1) = e - e^2$

$(\because y = e \text{ at } x = 1)$

10.5 **(2)**

$$f(1) + \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \ldots\ldots + \frac{f^n(1)}{n!} = 1 + \frac{n}{1!} + \frac{n(n-1)}{2!} + \ldots. + \frac{n!}{n!}$$

$$= {}^nC_0 + {}^nC_1 + {}^nC_1 + \ldots\ldots + {}^nC_n$$

$$= 2^n$$

10.6 **(4)**

$y\ \ell n \sin x = x\ \ell n \cos y$

$y'\ \ell n\sin x + y \cot x = \ell n\cos y - xy' \tan y$

$$y' = \frac{\ell n\cos y - y\cot x}{\ell n\sin x + x\tan y}$$

10.7 **(1)**

$$\frac{dy}{dx} = \cos x + e^x$$

$$\frac{d^2y}{dx^2} = -\sin x + e^x$$

$$\frac{d^2x}{dy^2} = -\frac{d^2y}{dx^2}\cdot\left(\frac{dx}{dy}\right)^3$$

$= (\sin x - e^x)\ (\cos x + e^x)^{-3}$

10.8 **(2)**

S_1 : $f(x) = -(x^2 + 5x + 6)$, if $-3 \le x \le -2$

$f'(x) = -2x - 5$

$$f'\left(\frac{-5}{2}\right) = 0$$

S_2 : $y = \sin^{-1}(\cos \pi x) = \frac{\pi}{2} - \cos^{-1}(\cos \pi x) = \begin{bmatrix} \frac{\pi}{2} - \pi x & 0 \le x \le 1 \\ \pi x - \frac{3\pi}{2} & 1 < x \le 2 \end{bmatrix}$

function is not differentiable at $x = 1$

hence no tangent can be drawn at this point

S_3 : $\frac{dx}{dt} = f'(t),\ \frac{dy}{dt} = g'(t)$

$$\frac{dx}{dy} = \frac{f'(t)}{g'(t)}$$

$$\frac{d^2x}{dy^2} = \frac{d}{dt}\left(\frac{f'(t)}{g'(t)}\right)\cdot\frac{dt}{dy}$$

$$= \frac{g'(t)f''(t) - g''(t)f'(t)}{(g'(t))^3}$$

10.9 (3)

S_1 : $3x^2 + 2yy' = 5(y + xy')$

at (1,3) $\quad 3 + 6y' = 5(3 + y')$

$\Rightarrow \quad y' = 12$

S_2 : $x^e = e^y$

$e\ell nx = y$

$\Rightarrow \quad y' = \frac{e}{x}$

at $\quad x = 1,\ y' = e$

S_3 : $f(x) = ax + b \quad \Rightarrow \quad f(\sin x) = a \sin x + b$

$f'(\sin x).\cos x = a \cos x \Rightarrow f'(\sin x) = a$

$f''(\sin x) = 0$

$\Rightarrow \quad f''(\sin x) + f'(\sin x) = b$ (constant)

10.10 (3)

$f(x) = x^n$

$f(3) = 1 \quad \Rightarrow \quad n = 0$

so $f(x) = 1 \quad \Rightarrow \quad f'(x) = 0$

10.11 (2)

$$\frac{dx}{dt} = -\sin t,\ \frac{dy}{dt} = \frac{1}{t}$$

$$\frac{dy}{dx} = \frac{-1}{t\sin t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx}$$

$$= \left(\frac{\sin t + t\cos t}{t^2 \sin^2 t}\right).(-\text{cosec } t)$$

$$\Rightarrow \quad \text{at } t = \frac{\pi}{2},\ \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\frac{4}{\pi^2} + \frac{4}{\pi^2} = 0$$

10.12 (3)

At $x = \frac{2\pi}{3}$, $\cos x < 0$ and $\sin x > 0$

$$y = -\cos x + \sin x,\ \frac{dy}{dx} = \sin x + \cos x$$

$$\frac{dy}{dx} \text{ at } x = \frac{2\pi}{3} \text{ is } = \sin\frac{2\pi}{3} + \cos\frac{2\pi}{3} = \frac{\sqrt{3}-1}{2}$$

10.13 (1)

$$\frac{dy}{dx} = f'\left(\frac{3x+4}{5x+6}\right)\frac{d}{dx}\left(\frac{3x+4}{5x+6}\right) = \tan\left(\frac{3x+4}{5x+6}\right)^2\left(\frac{-2}{(5x+6)^2}\right)$$

10.14 (4)

$$y = \tan^{-1}\left(\frac{2^{x+1}-2^{x}}{1+2^{x}.2^{x+1}}\right) = \tan^{-1} 2^{x+1} - \tan^{-1} 2^{x}$$

$$\frac{dy}{dx} = \frac{1}{1+(2^{x+1})^2}.2^{x+1}\log_e 2 - \frac{1}{1+(2^{x})^2}.2^{x}\log_e 2$$

$$\frac{dy}{dx} \text{ at } x = 0 \text{ is } = \frac{2\log_e 2}{5} - \frac{\log_e 2}{2}$$

$$\frac{dy}{dx} \text{ at } x = 0 \text{ is } = \frac{-\log_e 2}{10}$$

10.15 (2)

$$y = \frac{1}{1+x^n/x^m+x^p/x^m} + \frac{1}{1+x^m/x^n+x^p/x^n} + \frac{1}{1+x^m/x^p+x^n/x^p}$$

$$= \frac{x^m}{x^m+x^n+x^p} + \frac{x^n}{x^p+x^m+x^n} + \frac{x^p}{x^p+x^m+x^n}$$

$$y = 1, \ \frac{dy}{dx} = 0$$

10.16 (2)

There exist no value of y which satisfy the given equation at x = e. Hence function is not defined at x = e

so $\frac{dy}{dx}$ will not exist at this point.

10.17 (3)

$$x = \cos^2\theta \qquad \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$\frac{dx}{d\theta} = -\sin 2\theta$$

$y = 2\sin^{-1}\sin\theta + \sin^{-1}(\sin 2\theta)$

$= 2\theta + \pi - 2\theta = \pi$

$$\frac{dy}{d\theta} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = 0$$

10.18 (1)

$h'(x) = 2f(x)f'(x) + 2g(x).g'(x)$

$= 2f'(x)\,(f(x)+g'(x))$

$= 2f'(x)\,(f(x) + f''(x))$

$= 0$

$\Rightarrow$ h(x) is a constant function

$\Rightarrow$ $h(10) = h(0) = 1$

10.19 **(4)**

$\ell ny = (\ell nx)^2 + \ell n(\tan^{-1} x)$

$$\frac{y'}{y} = \frac{2\ell nx}{x} + \frac{1}{(\tan^{-1}x)(1+x^2)}$$

at $x = 1$, $y = \frac{\pi}{4}$ $\Rightarrow$ $y' = \frac{1}{2}$

10.20 **(4)**

$y = e^{xy}$ $\Rightarrow$ $\ell ny = xy$

$\frac{y'}{y} = y + xy'$ $\Rightarrow$ $y' = \frac{y^2}{1-xy} = \frac{e^{2xy}}{1-xy}$

10.21 **(2)**

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{2x+5y-2}{5x+2y+1} = \frac{-5}{8} \text{ at } (1, 1)$$

10.22 **(1)**

$$\ell ny = \ell n(1 + x) + \ell n(1 + x^2) + \ell n\,(1 + x^4) + ... + \ell n\,(1+x^{2^n})$$

$$\frac{y'}{y} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{(1+x^4)} + ... + \frac{2^n x^{2^n-1}}{(1+x^{2^n})}$$

$\Rightarrow$ $y'(0) = 1$

10.23 **(1)**

$$\frac{d}{dx}\ell n\,|x| = \frac{d}{dx}\ell n(-x) = \frac{1}{(-x)}(-1) = \frac{1}{x}$$

10.24 **(4)**

$y = x^2$

$$\frac{dy}{dx} = 2x$$

$$\frac{d^2y}{dx^2} = 2$$

again $2x\,\frac{dx}{dy} = 1$

$$2\left(\frac{dx}{dy}\right)^2 + 2x\,\frac{d^2x}{dy^2} = 0 \Rightarrow x\,\frac{d^2x}{dy^2} = -\left(\frac{dx}{dy}\right)^2 \Rightarrow \frac{d^2x}{dy^2} = -\frac{1}{4x^2} \Rightarrow \left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right) \neq 1$$

Statement-2 :

$$\because \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} \Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{\left(\frac{dx}{dy}\right)^2}.\frac{d^2x}{dy^2}.\frac{dy}{dx} = -\left(\frac{dy}{dx}\right)^3.\frac{d^2x}{dy^2}$$

11. APPLICATION OF DERIVATIVES

11.1 (2)
f(x) is not differentiable at x = 2 and $x = n\pi, n \in I$

11.2 (2)
Given $f(x) = a\log x + bx^2 + x$, $x > 0$

$f'(x) = \frac{a}{x} + 2bx + 1$

since f(x) has extremum at x = 1 and x = 3
$\therefore$ $f'(1) = 0$ and $f'(3) = 0$
$a + 2b + 1 = 0$ and $a + 18b + 3 = 0$

$a = -\frac{3}{4}$, $b = -\frac{1}{8}$

11.3 (2)
Here $f(x) = x^3 + (a + 2)x^2 + 3ax + 5$
$\therefore$ $f'(x) = 3x^2 + 2(a + 2)x + 3a$
$\forall x \in R$, $f'(x) > 0 \Rightarrow 3x^2 + 2(a + 2)x + 3a > 0$
$\Rightarrow 4(a + 2)^2 - 36a < 0$
$\Rightarrow a^2 + 4a + 4 - 9a < 0$
$\Rightarrow a^2 - 5a + 4 < 0$
$\Rightarrow a \in (1, 4)$
$\therefore$ f(x) is M.I. for $a \in (1, 4)$
$\therefore$ f(x) is invertible for $a \in (1, 4)$

Now a = 1 then $f(x) = (x + 1)^3 + 4 \Rightarrow x = \sqrt[3]{y-4} - 1$ which is defined for all $y \in R$

for all $y \in R$
when a = 4 then $y = f(x) = (x + 2)^2 - 3$

$\Rightarrow x = -2 + \sqrt[3]{y+3}$ which is defined for all $y \in R$

Hence f(x) is invertible for $a \in [1, 4]$

11.4 (3)

$\frac{dy}{dx} = x^3 - 3x + \lambda$

We must have $\frac{dy}{dx} = 0$ for three values

$\Rightarrow$ Equation $x^3 - 3x + \lambda = 0$ has three real roots.
Let $g(x) = x^3 - 3x + \lambda$
then $g'(x) = 0$ for $x = 1, -1$
For three roots of $g(x) = 0$
$g(1)\, g(-1) < 0$
$\Rightarrow -2 < \lambda < 2 \Rightarrow k = 2$

11.5 (3)

$$f(x) = \begin{cases} x^{\alpha} \sin\frac{\pi}{nx} & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}, \ n \in I, n \neq 0$$

$f(0) = 0$ $\quad\therefore\quad$ $f(1) = \sin\frac{\pi}{n} = 0$ $\quad\Rightarrow\quad$ $n = -1, 1$

$\therefore$ greatest value of n is 1 and least value of n is – 1
f(x) is continuous at x = 0 iff $\alpha > 0$
f(x) is continuous at x = 1 (always)

11.6 (3)

For vertical tangents $\frac{dx}{d\phi} = 0$

so $-3\cos\phi = 0$ $\quad\Rightarrow\quad$ $\phi = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

at $\quad \phi = \frac{\pi}{2}$, x = – 1, y = 3 so (–1, 3)

$\phi = \frac{3\pi}{2}$, x = 5, y = 3 so (5, 3)

11.7 (3)

$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta) = a\,\theta\cos\theta$

$\frac{dy}{d\theta} = a\theta\sin\theta$

$\frac{dy}{dx} = \tan\theta$

equation of normal $x\cos\theta + y\sin\theta = a$

whose distance from origin is $\left|\frac{0+0-a}{\sqrt{\cos^2\theta + \sin^2\theta}}\right| = a$

11.8 (2)

Differentiating the equation of curve xy = 1

we have $\frac{dy}{dx} = -\frac{y}{x}$

hence slope of normal = $\frac{x}{y}$. Moreover the slope of line Ax + By + C = 0 is $-\frac{A}{B}$

so $\frac{x}{y} = -\frac{A}{B}$

Bx + Ay = 0, solve it with xy = 1

we have $x^2 = -\frac{A}{B}$, so we must have $\frac{A}{B} < 0$.

11.9 (2)

$$PT = |k| \sqrt{1+\frac{1}{m^2}}, \; PN = |k| \sqrt{1+m^2}, \text{ where } m = f'(x), k = \text{constant}$$

$$TM = \left|\frac{k}{m}\right|, \; MN = |km|$$

11.10 (4)

(Velocity) $V = \frac{ds}{dt} = t^2 - 16 = 0 \Rightarrow t = \pm 4$ and acceleration = 2t = 2.4 = 8

11.11 (2)

$$f'(c) = \frac{f(x) - f(0)}{x - 0} \quad c \in (0, 2)$$

$$\frac{f(x)}{x} \le \frac{1}{2} \qquad x \in [0, 2] \Rightarrow \frac{1}{x} \ge \frac{1}{2}$$

$f(x) \le 1$

11.12 (1)

$f'(x) = 3ax^2 + 2bx + 11 = 0$

$$x = \frac{-2b \pm \sqrt{4b^2 - 4.3a.11}}{2 \times 3a} = 2 + \frac{1}{\sqrt{3}}$$

a = 1, b = – 6

11.13 (1)

$$f(x) = \frac{1}{x} \quad \Rightarrow \quad f'(x) = -\frac{1}{x^2}$$

$f(b) - f(a) = (b - a) f'(x_1)$

$$\frac{1}{b} - \frac{1}{a} = (b - a)\left(-\frac{1}{x_1^2}\right), \; a < x_1 < b$$

$x_1 = \sqrt{ab}$

11.14 (2)

Differentiating the function $f(x) = \log x - (x - 1)$

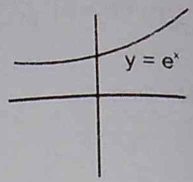
we get $f'(x) = \frac{1}{x} - 1 = \frac{1-x}{x} < 0$ for $x \in (1, \infty)$

f(x) is decreasing, $f(1) > f(x)$

11.15 (1)

$f'(x) = 2x + 12 > 0, \; x \in [-1, 2]$

11.16 (4)



y = e^x is increasing and concave upward $\frac{e^{x_1}+e^{x_2}}{2} > e^{\frac{x_1+x_2}{2}}$

and $\frac{e^{x_1}+e^{x_2}+e^{x_3}}{3} > e^{\left(\frac{x_1+x_2+x_3}{3}\right)}$

11.17 (1)

$f'(1) = a^2 + 3a + 2 < 0$

$a \in (-2, -1)$

11.18 (3)

$f'(x) = 2 \sin x \cos x$

$f'(x) = \sin 2x$

$f''(x) = \cos 2x \cdot 2$

$f''(x) = 0 \quad \Rightarrow \quad 2x = \frac{\pi}{2}, \frac{3\pi}{2}, x = \frac{\pi}{4}, \frac{3\pi}{4}$

$f'''\left(\frac{3\pi}{4}\right) = -4 \sin 2\left(\frac{3\pi}{4}\right) \neq 0$

11.19 (3)

$f'(x) = x(e^x - 1)(x - 1)(x - 2)^3 (x - 5)^5 = 0$

local maxima at x = 2

11.20 (2)

Let the line $\frac{x}{a} + \frac{y}{b} = 1$ it passes through (h, k)

$\therefore \quad \frac{h}{a} + \frac{k}{b} = 1 \quad \Rightarrow \quad b = \frac{ak}{a-h}$

$\Delta = \frac{1}{2} ab = \frac{1}{2} a \frac{ak}{(a-h)}$

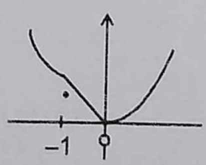
$\Delta' = 0$ at $a = 2h$

$\Delta_{max} = \frac{1}{2} \cdot 4h^2 \cdot \frac{k}{h} = 2hk$

11.21 (2)

For minima at x = 0

$p \geq 0, k > 0$



1.22 (2)

$V = \pi x^2 y$

$\frac{y}{r-x} = \frac{h}{r}$

$V = \pi x^2 \left(\frac{h}{r}(r-x)\right)$

$V' = 0 = x = \frac{2r}{3}$

1.23 (2)

$xy^n = a^{n+1} \quad \Rightarrow$

length of subnormal

$n + 2 = 0$

$n = -2$

11.24 (3)

$f'(x) = 3\lambda x^2 - 4\lambda x +$

$D < 0$ and $3\lambda > 0$

$16\lambda^2 - 12\lambda(\lambda + 1)$

$16\lambda^2 - 12\lambda^2 - 12\lambda$

$4\lambda^2 - 12\lambda < 0$

$\lambda^2 - 3\lambda < 0$

$\lambda \in (0, 3)$

11.25 (2)

$\sec^{-1} x$ is incre

11.26 (2)

By LMVT the

such that f(6

$\frac{f(6)+2}{5} =$

$f(6) = 5f'$ (3

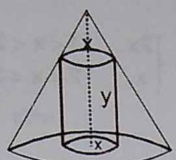
$f(6) \geq 8$

1.22 **(2)**

$V = \pi x^2 y$

$\frac{y}{r-x} = \frac{h}{r}$

$V = \pi x^2 \left(\frac{h}{r}(r-x)\right)$



$V' = 0 = x = \frac{2r}{3}$

11.23 **(2)**

$xy^n = a^{n+1}$ $\Rightarrow$ $\log x + n \log y = (n+1) \log a$

$\frac{1}{x} + \frac{n}{y}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{y}{nx}$

length of subnormal $= y \cdot \frac{dy}{dx} = -\frac{y^2}{nx} = \frac{-y^2y^n}{na^{n+1}} = \frac{-y^{n+2}}{na^{n+1}}$

$n + 2 = 0$

$n = -2$

11.24 **(3)**

$f'(x) = 3\lambda x^2 - 4\lambda x + (\lambda + 1) > 0$

$D < 0$ and $3\lambda > 0$

$16\lambda^2 - 12\lambda(\lambda + 1) < 0, \lambda > 0$

$16\lambda^2 - 12\lambda^2 - 12\lambda < 0$

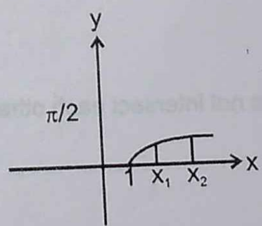
$4\lambda^2 - 12\lambda < 0$

$\lambda^2 - 3\lambda < 0$

$\lambda \in (0, 3)$

11.25 **(2)**

$\sec^{-1} x$ is increasing and concave down ward.



11.26 **(2)**

By LMVT there is $c \in (1, 6)$

such that $\frac{f(6) - f(1)}{6-1} = f'(3)$

$\frac{f(6)+2}{5} = f'(3)$

$f(6) = 5f'(3) - 2 \geq 5.2 - 2$

$f(6) \geq 8$

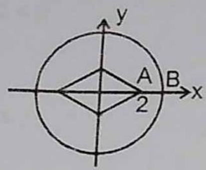
11.27 (2,4)

$$f(x) = \begin{cases} x^2+2 & , \ 1 \le x < 2 \\ \dfrac{x^2+2}{2} & , \ 2 \le x < 3 \\ \dfrac{11}{3} & , \ x = 3 \end{cases} \Rightarrow \quad f'(x) = \begin{cases} 2x & , \ 1 < x < 2 \\ x & , \ 2 < x < 3 \end{cases}$$

function is increasing in each of the two intervals (1, 2) and (2, 3) but not in [1, 3]
Clearly least value of f(x) is 3 and greatest value of f(x) does not exist.

11.28 (2)

AB = 2



11.29 (2)

$$\frac{dy}{dx} = 3x^2 + 2x - 1 = 0$$

$$x = -1 \text{ or } x = \frac{1}{3}$$

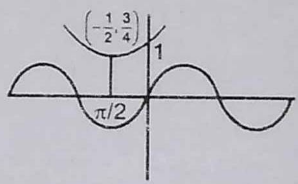
when $x = -1$, $y = 1$, $x = \frac{1}{3}$, $y = -\frac{5}{27}$

$$\text{distance} = 1 + \frac{5}{27} = \frac{32}{27}$$

11.30 (1)

$$x^2 + x + 1 \in \left[\frac{3}{4}, \infty\right)$$

sin x and $x^2 + x + 1$ does not intersect each other



11.31 (3)

$V = \pi r^2 h$

Let k be the thickness of sides then that of the top will be $\frac{5}{4}$ k.

$$\therefore \quad S = k\left(\frac{2v}{r} + \frac{5}{4}\pi r^2\right)$$

$$\frac{dS}{dr} = k\left(-\frac{2v}{r^2} + \frac{5}{2}\pi r\right) \text{ and } \frac{d^2s}{dr^2} = k \cdot \left(\frac{4v}{r^3} + \frac{5}{2}\pi\right) > 0 \text{ for positive r}$$

$$\frac{dS}{dr} = 0 \text{ will give } \frac{r}{h} = \frac{4}{5}$$

2 (4)

Time taken by boat = $\frac{300}{x}$ hours

petrol consumed = $\left(2+\frac{x^2}{600}\right)\frac{300}{x}$ liter

expenses on travelling

$$E = 200 \times \frac{300}{x} + \left(2+\frac{x^2}{600}\right)\frac{3000}{x} = \frac{60000}{x} + \frac{6000}{x} + 5x = \frac{66000}{x} + 5x$$

$$\frac{dE}{dx} = \frac{-66000}{x^2} + 5 < 0 \text{ for all } [25, 60]$$

most economical speed is 60 kmph.

.33 (4)

$f'(x) = 2x\,(a_1 + 2a_2 x^2 + ... + na_n x^{2n-2})$

$f'(x) = 0 \quad \Rightarrow \quad x = 0$

$f''(x) = 2\,(a_1 + 6a_2 x^2 + ... + n\,(2n-1)\,a_n x^{2n-2})$

$(f''(x))_{x=0} = 2a_1 > 0$

P(x) has only minimum at x = 0

1.34 (4)

Let point is $(r\cos\theta, r\sin\theta)$ whose distance from origin is r.

It will lie on curve.

$2r^2\cos^2\theta + 5r^2\cos\theta\sin\theta + 2r^2\sin^2\theta = 1$

$2r^2 + \frac{5r^2}{2}\sin 2\theta = 1$

$4r^2 + 5r^2\sin 2\theta = 2$

$r^2 = \frac{2}{4+5\sin 2\theta}$

$r^2 = \frac{2}{9}$ [minimum when $\sin 2\theta = 1$]

$r = \frac{\sqrt{2}}{3}$

11.35 (2)

$f(a) = 1,\ f(b) = 1,\ f(c) = 1$

$\Rightarrow \quad f(x) = 1$ is an identity. $\quad \Rightarrow \quad \frac{d}{dx} f(x) = 0$

$f(1) = 1,\ f(2) = 1,\ f(3) = 1$

11.36 (1)

Using LMVT for f in [1, 2] $\forall$ c $\in$ (1, 2), $-\ \frac{f(2)-f(1)}{2-1} = f'(3) \le 2,\ f(2) - f(1) \le 2$ ...(i)

$\Rightarrow$ f(2) $\le$ 4 again LMVT in [2, 4] $\forall$ d $\in$ [2, 4], $\frac{f(4)-f(2)}{4-2} = f'(4) \le 2$

$f(4) - f(2) \le 4$
$8 - f(2) \le 4$
$f(2) \ge 4$ ....(ii)

from (i) and (ii) f(2) = 4

11.37 (3)

$y = x^4$
equation of tangent at (α, α^4) is
$y - \alpha^4 = 4\alpha^3 (x - \alpha)$

since it passes through the point $\left(\frac{3}{4}, 0\right)$

$\therefore$ $-\alpha^4 = 3\alpha^3 - 4\alpha^4$ $\Rightarrow$ $\alpha = 0, 1$

$\therefore$ there are two tangents

11.38 (4)

$$f(x) = x^{\frac{1}{3}} . (x-1)^{\frac{2}{3}}$$

$$f'(x) = \frac{(x-1)^{2/3}}{3 . x^{2/3}} + \frac{2x^{1/3}}{3(x-1)^{1/3}} = \frac{3x-1}{3 . x^{2/3} . (x-1)^{1/3}}$$

$\therefore$ the critical points are $0, \frac{1}{3}, 1$

11.39 (1)

Greatest value of f is $\frac{\pi}{2}$ and least value is 0.

11.40 (3)

f decreases on $(-\infty, 0)$ and increases on $(0, \infty)$

11.41 (1)

In the [–1, 0), then function increases from $\frac{1}{e} + 1$ to 2 and in (0, 1] it increases from 0 to e – 1, it does not attain either the value 2 or 0, therefore the function is bounded but never reaches its maximum and minimum. This is because there is a discontinuity at the point x = 0.

11.42 (4)

$$\int_{0} \log \sin x \, dx - \int_{0} \log \cos x \, dx$$

$$= \int_{0}^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx - \int_{0}^{\pi/2} \log \cos x \, dx$$

$$= \int_{0}^{\pi/2} \log \cos x \, dx - \int_{0}^{\pi/2} \log \cos x \, dx = 0$$

13.4 **(2)**

$$I = \int_{0}^{5\pi} [\tan^{-1} x] dx$$

$$= \int_{0}^{\tan 1} [\tan^{-1} x] dx + \int_{\tan 1}^{5\pi} [\tan^{-1} x] dx$$

$$= 0 + \int_{\tan 1}^{5\pi} . dx = (5\pi - \tan 1)$$

13.5 **(2)**

$$I = \int_{-1}^{1} \frac{x^3 dx}{x^2 + 2|x| + 1} + \int_{-1}^{1} \frac{|x|+1}{x^2 + 2|x| + 1} dx$$

$$= 0 + 2\int_0^1 \frac{|x|+1}{(|x|+1)^2} dx = 2\int_0^1 \frac{dx}{1+x} = (2\ell n|1+x|)_0^1 = 2\ell n2$$

13.6 **(3)**

$$I = \int_{-\pi/4}^{\pi/4} \frac{x^{11} - 3x^9 + 8x^3 - 4x + 1}{\sec^2 x} dx + \int_{-\pi/4}^{\pi/4} \sec^2 x dx$$

$$I = 0 + 2\int_0^{\pi/4} \sec^2 x dx = 2(\tan x)_0^{\pi/4} = 2$$

13.7 **(2*)**

$$I = \int_0^{\pi/4} \left(\frac{\sec^2 x}{1+3^x} + \frac{\sec^2(-x)}{1+3^{-x}} \right) dx$$

$$= \int_0^{\pi/4} \sec^2 x dx = (\tan x)_0^{\pi/4} = 1$$

13.8 **(2)**

$f(x) = (1 + a) \sin x,$

$$a = \int_0^{\pi/2} f(t)\cos t\, dt = (1 + a) \int_0^{\pi/2} \sin t \cos t\, dt$$

$$= \frac{1+a}{2} \Rightarrow a = 1, \quad f(x) = 2 \sin x.$$

$$\int_0^{\pi/2} f(x) dx = 2\int_0^{\pi/2} \sin x dx = 2.$$

13.9 **(1)**

$$f'(x) = -[6 \sin x + 2 \cos x] > 0 \quad \forall x \in \left[\frac{5\pi}{3}, \frac{7\pi}{4}\right]$$

$\Rightarrow$ f(x) is an increasing function for $x \in \left[\frac{5\pi}{3}, \frac{7\pi}{4}\right]$

$$\text{Greatest value} = f\left(\frac{7\pi}{4}\right) = \int_{5\pi/3}^{7\pi/4} (6\cos t - 2\sin t) dt$$

$$= (6\sin t + 2\cos t)_{5\pi/3}^{7\pi/4} = 3\sqrt{3} - 2\sqrt{2} - 1$$

13.10 (4)

$$f(x) = \int_0^x f(t)dt,\ f(0) = 0$$

$\therefore \quad f'(x) = f(x)$

$f(x) = Ae^x$

$f(0) = 0 \quad \Rightarrow \quad A = 0$

$\therefore \quad f(x) = 0$

13.11 (4)

$$L = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \left(\left(\frac{r}{n}\right)^{\frac{1}{a}} + \left(\frac{r}{n}\right)^{a} \right)$$

$$= \int_0^1 \left(x^{\frac{1}{a}} + x^a \right) dx = \frac{a}{a+1} + \frac{1}{a+1} = 1$$

13.12 (2)

$$\int_0^{\pi} \cos^n x\, dx = 2 \int_0^{\pi/2} \cos^{2k} x\, dx \text{, where } n = 2k$$

$$= 2 \cdot \frac{2k-1}{2k} \cdot \frac{2k-3}{2k-2} \cdots \frac{1}{2} \cdot \frac{\pi}{2}$$

13.13 (2)

$$I = \int_0^{\pi/2} 2\sin x \tan^{-1}(\sin x)\cos x\, dx$$

On putting $\sin x = t$ we get

$\sin x = t$

$$I = \int_0^1 2t \tan^{-1} t\, dt$$

$$= \left[\tan^{-1} t . t^2 - \int \frac{t^2}{1+t^2} dt \right]_0^1$$

$$= \left[t^2 \tan^{-1} t - t + \tan^{-1} t \right]_0^1$$

$$= \frac{\pi}{2} - 1$$

13.14 (4)

$$\text{sgn}(x - [x] = \begin{cases} 1, & x \notin I \\ 0, & x \in I \end{cases}$$

$$\int_{-2}^{5} \text{sgn}(x - [x] dx = 7\int_0^1 1.dx = 7$$

13.15 (3)

$$I = \int_0^{18} \frac{[x^2]dx}{[x^2 - 36x + 324] + [x^2]} \quad \text{.... (i)}$$

$$I = \int_0^{18} \frac{[(18-x)^2]dx}{[x^2] + [(18-x)^2]} \quad \text{.... (ii)}$$

Adding

$$2I = \int_0^{18} 1.dx = 9$$

13.16 (2)

$$\int_a^0 (9^{-2t} - 2(9)^{-t})dt \le 0$$

$$\left(\frac{9^{-2t}}{-2\ell n9} - \frac{2.9^{-t}}{-\ell n9}\right)_a^0 \le 0$$

$$\left(-9^{-2t} + 4(9)^{-t}\right)_a^0 \le 0$$

$9^{-2a} - 49^{-a} + 3 \le 0 \qquad \{9^{-a} = t\}$

$t^2 - 4t + 3 \le 0$

$(t-1)(t-3) \le 0$

$1 \le 9^{-a} \le 3$

$3^0 \le 3^{-2a} \le 3^1$

$0 \le -2a \le 1$

$0 \ge a \ge -\frac{1}{2}$

So $\quad a = 0$ only

13.17 (1)

$\because \quad (-1)^{[x]}$ is odd if $x \notin$ integer

$$\therefore \quad \int_{-n}^{n} (-1)^{[x]}dx = 0$$

13.18 (1)

$F'(x) = 2(x+3)(5-x-3) - 2x(5-x)$

$= 12(1-x)$

$F'(x) = 0 \quad \Rightarrow \quad x = 1$

$$\therefore \quad M = \int_1^4 2t(5-t)dt = 75 - 42 = 33$$

13.19 (2)

$$\int_0^{\pi/4} \frac{\sin^2 x}{\sin^2 2x}\left(\frac{1}{1-e^{-3x}} + \frac{1}{1+e^{3x}}\right)dx$$

$$= \frac{1}{4}\int_0^{\pi/4} \sec^2 dx = \left(\frac{1}{4}\tan x\right)_0^{\pi/4} = \frac{1}{4}$$

13.20 (1)

$$I = \int_{-1}^{1}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx + \int_{1}^{3}\frac{\pi}{2}dx = (3-1)\ \frac{\pi}{2} = \pi$$

13.21 (1)

$$\int_0^1 (C_2x^2 + C_1x + C_0)\,dx$$

$$= \left[\frac{C_2x^3}{3}+\frac{C_1x^2}{2}+C_0x\right]_0^1 = \frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3} = 0 \text{ (given)}$$

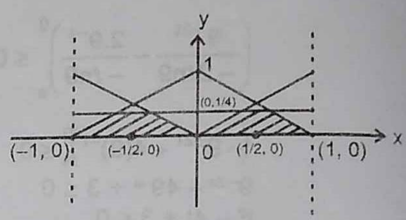
$\Rightarrow$ graph $y = C_2x^2 + C_1x + C_0$ crosses x-axis atleast once.

$\Rightarrow$ at least one root of the equation $C_2x^2 + C_1x + C_0 = 0$ is present in (0, 1)

13.22 (2)

$$\int_{-1}^{1} f(x)\,dx = 2\int_{0}^{1} f(x)\,dx = 2\left(\int_0^{1/4} x\,dx + \int_{1/4}^{3/4}\frac{1}{4}dx + \int_{3/4}^{1}(1-x)\,dx\right)$$

$$= 2\left(\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}\right) = \frac{3}{8}$$



13.23 (3)

$$I_2 = \int_0^1 \frac{x^2dx}{e^{x^3}(2-x^3)}$$

Let $1 - x^3 = t$

$\Rightarrow$ $-3x^2dx = dt$

$$\Rightarrow\quad I_2 = \frac{1}{3}\int_0^1 \frac{dt}{e^{1-t}(1+t)} = \frac{1}{3e}\int_0^1\frac{e^t dt}{1+t} = \frac{I_1}{3e}$$

$$\Rightarrow\quad \frac{I_1}{I_2} = 3e.$$

13.24 (2)

Let $n \le x < n + 1$, where $n \in I,\ I > 0$

$$I = \int_0^x 2^{\{t\}}dt = \int_0^n 2^{\{t\}}dt + \int_n^x 2^{\{t\}}dt = n\left(\frac{2^t}{\ell n2}\right)_0^1 + \left(\frac{2^t}{\ell n2}\right)_n^x = \frac{1}{\ell n2}\left(2^x - 2^{[x]} + [x]\right)$$

13.25 (3)

$$2f(x)\,.\,f'(x) = f(x)\ .\ \frac{\cos x}{2+\sin x} \qquad f(0) = 0$$

$$2f'(x) = \frac{\cos x}{2+\sin x}$$

Integrating on both side

$$2f(x) = \int\frac{\cos x\,dx}{2+\sin x}$$

$2f(x) = \ell n\,(2 + \sin x) + c$

$x = 0 \quad \Rightarrow \quad c = -\ell n\,2$

$$f(x) = \frac{1}{2}\ell n\left(\frac{2+\sin x}{2}\right),\quad x \ne n\pi,\quad n \in I$$

13.26 (4)

$$f(1) = \int_0^1 \frac{x^{\cos\alpha} - 1}{\ell n x} dx$$

$$\frac{df}{d\alpha} = \int_0^1 (-\sin\alpha) \cdot x^{\cos\alpha}.dx = -\left(\sin\alpha.\frac{x^{\cos\alpha+1}}{\cos\alpha+1}\right)_0^1 = \frac{-\sin\alpha}{1+\cos\alpha}$$

Integrating $f(1) = \ell n\ (1 + \cos\alpha) + c$

$f\left(\frac{\pi}{2}\right) = 0 \quad \Rightarrow \quad c = 0 \quad \therefore \quad f(1) = \ln\ (1 + \cos\alpha)$

13.27 (4)

$$f(x) = x^2 + \int_0^x e^{-t} f(x-t)dt \qquad t \to x - t$$

$$f(x) = x^2 + \int_0^x e^{-(x-t)} f(t)dt \qquad \ldots\ldots (1)$$

$$f(x) = x^2 + e^{-x} \int_0^x e^t f(t)dt$$

$$f'(x) = 2x - e^{-x} \int_0^x e^t f(t)dt + e^{-x}.e^x f(x)$$

$$f'(x) = 2x + f(x) - e^{-x} \int_0^x e^t f(t)dt \qquad \ldots\ldots (2)$$

$(1) + (2) \Rightarrow f'(x) = x^2 + 2x$

$\Rightarrow \quad f(x) = \frac{x^3}{3} + x^2 + c, \ f(0) = 0 \quad \Rightarrow \quad c = 0$

$\Rightarrow \quad f(x) = \frac{x^3}{3} + x^2, \ f(1) = \frac{1}{3} + 1 = \frac{4}{3}$

13.28 (1)

$$I = \int_{-\pi/2}^{\pi/2} [\tan(0 - x)] dx$$

$$I = \int_{-\pi/2}^{\pi/2} [-\tan x] dx$$

$$2I = \int_{-\pi/2}^{\pi/2} ([\tan x] + [-\tan x]) dx$$

$$= \int_{-\pi/2}^{\pi/2} -1\, dx$$

$$2I = -(x)_{-\pi/2}^{\pi/2} = -\pi$$

$$I = -\pi/2$$

13.29 (1)

$$I = 4\int_0^{\pi/4} x\left(\frac{\pi}{4}-x\right)\ell n(1+\tan x)dx$$

$$I = 4\int_0^{\pi/4}\left(\frac{\pi}{4}-x\right)x\,\ell n\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$$

$$2I = 4\int_0^{\pi/4} x\left(\frac{\pi}{4}-x\right)\ell n2.dx$$

$$\ell = 2\ell n2\left[\left(\frac{\pi}{4}\right)^3\frac{1}{2}-\left(\frac{\pi}{4}\right)^3\frac{1}{3}\right]$$

$$I = \frac{\pi^3}{192}\ell n2$$

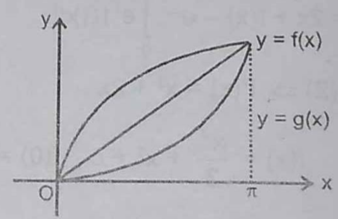
13.30 (3)

The curve $y = g(x)$ is the image of the curve $y = f(x)$ in the line $y = x$

$$\therefore \int_0^{\pi}\frac{f(x)+g(x)}{2}dx = \int_0^{\pi}x\,dx = \frac{\pi^2}{2}$$

$$\therefore \int_0^{\pi}g(x)\,dx = \pi^2 - \int_0^{\pi}f(x)\,dx$$

$$= \pi^2 - \int_0^{\pi}(x+\sin x)dx = \frac{\pi^2}{2}-2$$



13.31 (4)

$$I = \int_0^{\pi/4}\tan^{-1}\left(\frac{1}{\tan^2\theta-\tan\theta+1}\right)\sec^2\theta\,d\theta$$

$\tan\theta = t$

$$I = \int_0^{1}\tan^{-1}\left\{\frac{t-(t-1)}{1+t(t-1)}\right\}dt$$

$$= \int_0^{1}\{\tan^{-1}-\tan^{-1}(t-1)\}\,dt$$

$$= \int_0^{1}\tan^{-1}t\,dt + \int_0^{1}\tan^{-1}t\,dt = 2p$$

13.32 (1)

Let f(x) = log (1 + x) – x f(0) = 0

$f'(x) = \frac{1}{1+x} - 1 < 0$ for $x \in (0,1)$

f(x) is monotonically decreasing in (0, 1)

$\Rightarrow \ell n\,(1 + x) < x$ in (0, 1)

statement 2 is true.

Now, $\int \ell n(1+x)dx < \int xdx$, using statement 2.

$= \int_0^1 \ell n(1+x) < \frac{1}{2}$

Statement - 1 is true choice (1)

13.33 (1)

$f'(x) = (x^2 - x + 2)\,(x + 3)\,(x + 2)\,(x + 1)\,(x - 2)\,(x - 3)\,(x - 4)$

+	–	+	–	+	–	+	
	–3	–2	–1	2	3	4	

f(x) has maximum at x = – 3, –1, + 3

sum of values of x = – 3 + 3 – 1 = – 1.

13.34 (4)

$I_{11} = \int_0^1 (1-x^5)^{11}dx$

$= \int_0^0 (1-x^5)(1-x^5)^{10}dx$

$= I_{10} - \int_0^1 x^5(1-x^5)^{10}dx$

$= I_{10} + \int_0^1 \frac{x}{5}(1-x^5)^{10} \;\; (-5x^4)\,dx$

Integrating by parts.

$I_{11} = I_{10} + \left(\frac{x}{5}\frac{(1-x^5)^{11}}{11}\right)_0^{11} - \frac{1}{55}\int_0^1 (1-x^5)^{11}dx$

$= I_{10} - \frac{I_{11}}{55}$

$= (1 + \frac{1}{55})\,I_{11} = I_{10}$

$\frac{I_{10}}{I_{11}} = \frac{56}{55}$

13.36 (2)

$S_1 : \int_0^1 \frac{\ell nx}{1+x}dx$

$= (\ell nx.\ \ell n(1+x))_0^1 - \int_0^1 \frac{\ell n(1+x)}{x}dx$

$= -\int_0^1 \frac{\ell n(1+x)}{x}dx$

S_2 : we know if f(x) is an odd then f′(x) is an even

13.37 (4)

S_1 : $I = \int_{-2}^{0}\{(x+1)^5 + \tan^7(x+1) + 4\}dx$

Let x + 1 = t ⇒ dx = dt

$I = \int_{-1}^{1}(t^5 + \tan^7 t + 4)dt$

$= 2\int_0^1 4.dt = 8$

S_2 : Let $h(x) = \frac{f(\sin x)}{f(\cos x) + f(\sin^2 x)}$

$\ell n(-x) = \frac{-f(\sin x)}{f(\cos x) + f(\sin^2 x)}$

⇒ h(x) is an odd h(x)

So I = 0

14. AREA UNDER CURVE

14.1 (2)

The curve is the ellipse $\frac{x^2}{1}+\frac{y^2}{2}=1$

$A=\pi ab=\sqrt{2}\pi$

14.2 (1)

The curve is symmetric about x-axis -

$$A=2\int_0^1 y\,dx = 2\int_0^1 x\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=2\int_0^1 \frac{x+x^2}{\sqrt{1-x^2}}dx,\ \text{put } x=\sin\theta$$

$$=2\int_0^{\pi/2}(\sin\theta+\sin^2\theta)\,d\theta = 2[1+\pi/4] = \frac{\pi}{2}+2$$

14.3 (3)

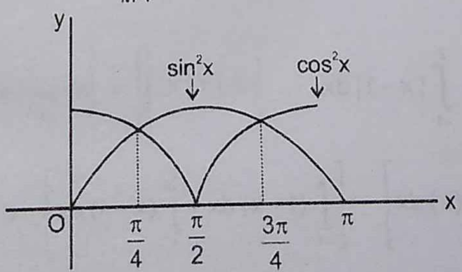
$$\frac{dy}{dx}=(1-2x^2)\,e^{-x^2}=0 \Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

y is maximum at $x=\frac{1}{\sqrt{2}}$

$$A=\int_0^{1/\sqrt{2}} xe^{-x^2}dx=\left[-\frac{1}{2}e^{-x^2}\right]_0^{1/\sqrt{2}}=\frac{1}{2}\left(1-e^{-1/2}\right)$$

14.4 (1)

$$\text{Required area}=\int_{\pi/4}^{3\pi/4}(\sin^2x-\cos^2x)dx=-\int_{\pi/4}^{3\pi/4}\cos 2x\,dx$$



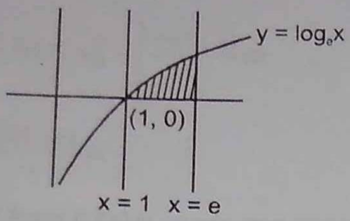
$$=-\left[\frac{\sin 2x}{2}\right]_{\pi/4}^{3\pi/4}$$

$$=-\frac{1}{2}\left(\sin\frac{3\pi}{2}-\sin\frac{\pi}{2}\right)$$

= 1 sq. unit

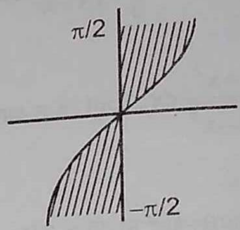
14.5 **(3)**

$$\text{Area} = \int_1^e y\,dx = \int_1^e \log_e x\,dx = [x\log_e x - x]_1^e$$



$= (e\log e - e) - (0 - 1) = 1$

14.6 **(1)**

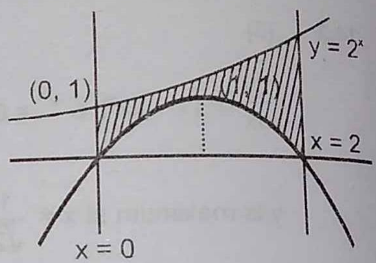


The required area $= 2\int_0^{\pi/2} x\,.\,dy$

$y = \sin^{-1}x \quad \Rightarrow \quad x = \sin y$

Required area $= 2\int_0^{\pi/2} \sin y\,.\,dy = -2\,[\cos y]_0^{\pi/2} = 2$

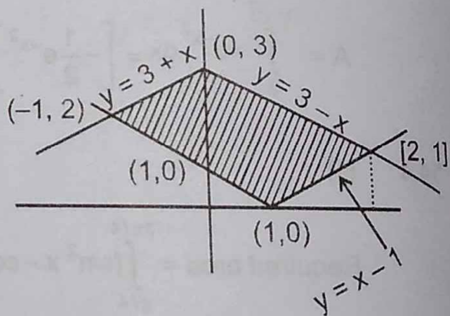
14.7 **(4)**



$$\text{Area} \int_0^2 (y_1 - y_2)\,.\,dx = \int_0^2 (2^x - (2x - x^2))\,.\,dx = \int_0^2 (2^x - 2x + x^2)\,.\,dx$$

$$\left[\frac{2^x}{\log_e 2} - x^2 + \frac{x^3}{3}\right]_0^2 \Rightarrow \left[\frac{4}{\log_e 2} - 4 + \frac{8}{3}\right] - \left[\frac{1}{\log_e 2}\right] = \frac{3}{\log_e 2} - \frac{4}{3}$$

14.8 **(1)**



$f_1(x) = |x - 1|, \quad f_2(x) = 3 - |x|$

$$f_1(x) = \begin{bmatrix} x - 1 & , & x \ge 1 \\ 1 - x & , & x < 1 \end{bmatrix}, \quad f_2(x) = \begin{bmatrix} 3 + x & , & x < 0 \\ 3 - x & , & x \ge 0 \end{bmatrix}$$

$$\text{Area} = \int_{-1}^{2} (f_2(x) - f_1(x))\,dx = \int_{-1}^{2} ((3 - |x|) - (|x - 1|))\,dx$$

$$\int_{-1}^{2} (3 - |x| - |x - 1|)\,dx$$

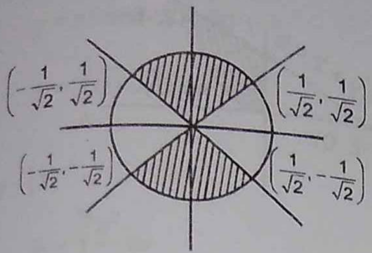
$$3[x]_{-1}^{2} - \int_{-1}^{2} |x|\,dx - \int_{-1}^{2} |x - 1|\,dx$$

$$3.3 - \left[\int_{-1}^{0} (-x)\,dx + \int_0^2 (x)\,dx\right] - \left[\int_{-1}^{1} (1 - x)\,dx + \int_1^2 (x - 1)\,dx\right]$$

$$9 - \frac{1}{2} - 2 + \left(-\frac{1}{2} - \frac{3}{2}\right) - \left(\frac{1}{2}\right)$$

$9 - 5 = 4$ sq. units

14.9 (3)



Required area = 4(area of shaded region in the first quadrant)

$$= 4\int_0^{\frac{1}{\sqrt{2}}} \left(\sqrt{1-x^2} - x\right) dx$$

$$A = 4\left[\frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - \frac{x^2}{2}\right]_0^{\frac{1}{\sqrt{2}}} = 4\left[\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{4}\right] = 4\left[\frac{1}{4} + \frac{\pi}{8} - \frac{1}{4}\right] = \frac{\pi}{2} \text{ sq. units}$$

14.10 (4)

$y = f(x)$ passes through (2, 0) and (0, 1)

$0 = f(2)$ and $f(0) = 1$

$$\int_0^2 f(x)\,dx = \frac{3}{4} \text{ given}$$

Now $$\int_0^2 xf'(x)\,dx = [xf(x)]_0^2 - \int_0^2 f(x)\,dx = 2f(2) - \int_0^2 f(x)\,dx = -\frac{3}{4}$$

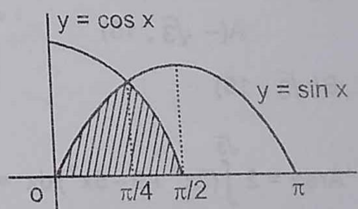
14.11 (2)

$$\text{Required area} = \int_0^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \cos x\,dx$$

$$= -\left[\cos x\right]_0^{\pi/4} + \left[\sin x\right]_{\pi/4}^{\pi/2}$$

$$= -\left[\frac{1}{\sqrt{2}} - 1\right] + \left[1 - \frac{1}{\sqrt{2}}\right] = 2\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$= \left(2 - \sqrt{2}\right) \text{ sq. units}$$

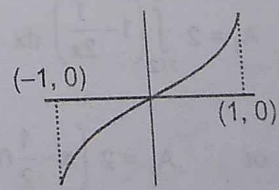


14.12 (2)

$$\text{Required area} = \left|\int_{-1}^{1} x|x|\,dx\right| = \left|\int_{-1}^{0} x|x|\,dx\right| + \left|\int_0^1 x|x|\,dx\right|$$

$$= \left|\int_{-1}^{0} -x^2\,dx\right| + \left|\int_0^1 x^2\,dx\right|$$

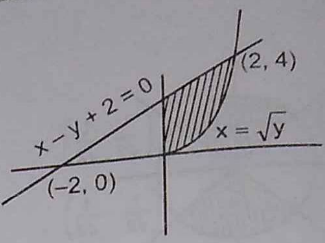
$$= \frac{1}{3} + \frac{1}{3} \quad \Rightarrow \quad \frac{2}{3} \text{ sq. units}$$



14.13 (4)

Required area $= \int_0^2 [(x+2)-(x^2)]\,dx = \int_0^2 (x+2-x^2)\,dx$

$= \left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_0^2 = 2+4-\frac{8}{3} = \frac{10}{3}$ sq. units



14.14 (1)

Required area $= \int_0^1 (\sqrt{x}-x)\,dx$

$A = \left[\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right]_0^1$

$A = \left[\frac{2}{3}-\frac{1}{2}\right] \Rightarrow \frac{1}{6}$ sq. units

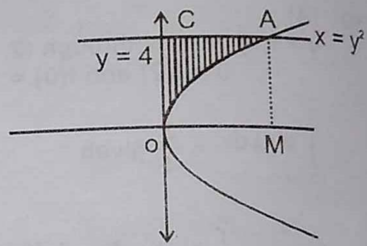
14.15 (2)

y = 4 meet the parabola $y^2 = x$ at A is (16, 4)

Required area = Area of rectangle OMAC – area of OMA

$= 4.16 - \int_0^{16} \sqrt{x}\,dx = 64 - \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^{16}$

$= 64 - \frac{128}{3} = \frac{64}{3}$ sq. units



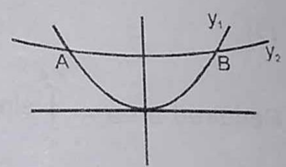
14.16 (1)

$y_1 = 5x^2$

$y_2 = 2x^2 + 9$

$A(-\sqrt{3}, 15)$

$B(\sqrt{3}, 15)$



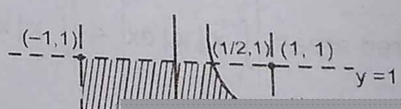
Figure

Area $= 2\int_0^{\sqrt{3}} (2x^2+9-5x^2)\,dx = 2\left[9x-x^3\right]_0^{\sqrt{3}}$

$= 2(9\sqrt{3}-3\sqrt{3}) = 12\sqrt{3}$

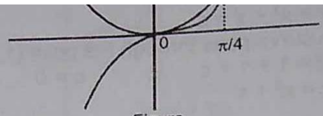
14.17 (2)

$A_1 = 2\int_{1/2}^{1} \left(1-\frac{1}{2x}\right) dx$



$$= \ell n|\sec x| - (\tan x - x)\Big]_0^{\pi/4}$$

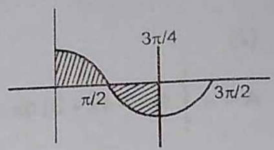
$$= \ell n\sqrt{2} - 1 + \frac{\pi}{4}$$



Figure

statement -2 is also true and it is correct explanation of statement -1

14.37 **(3)**



Figure

$$\text{Area} = \int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^{3\pi/4} \cos x\,dx$$

$= 2 - \frac{1}{\sqrt{2}}$ $\Rightarrow$ statement -1 is false

Also statement -2 is false (from theory).

15. Differential Equation

15.1 (1)

Given equation can be rewritten is $\frac{x\cos x - \sin x}{x^2}dx + ydx + xdy = 0$

$\Rightarrow d\left(\frac{\sin x}{x}\right) + d(xy) = 0$

Integrating , we get

$\frac{\sin x}{x} + xy = c$

15.2 (1)

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

$\Rightarrow \quad \frac{d}{dx}\left[y\frac{dy}{dx}\right] = 0 \Rightarrow y\frac{dy}{dx} = \text{Constant} = c$

at x = 0, y = 2, y′ = 4 $\Rightarrow$ c = 8

$\therefore y\frac{dy}{dx} = 8 \Rightarrow ydy = 8dx$

Intergrating both sides $\frac{y^2}{2} = 8x + d$

$\because y(0) = 2 \Rightarrow d = 2$

$\therefore y^2 = 16\left(x + \frac{1}{4}\right)$, Which is a parobola having focus at $\left[\frac{15}{4}, 0\right]$

15.3 (4)

The circles are $(x-h)^2 + (y-k)^2 = r^2$

Differentiating repeatedly

$x - h + (y-k)\,y_1 = 0$

$1 + y_1^2 + (y-k)y_2 = 0$ ....(i)

$3y_1y_2 + (y-k)\,y_3 = 0$ (ii)

Eliminate y – k from (i) , (ii)

$(1 + y_1^2)\,y_3 = 3y_1y_2^2$

15.4 (3)

Number of arbitrary constants in the equation are 3

hence order = 3

15.5 (3)

$\left[4 + \left(\frac{dy}{dx}\right)^2\right]^2 = \left(\frac{d^2y}{dx^2}\right)^3$

$\Rightarrow$ order =2, degree = 3

15.6 (1)

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

$$e^y \frac{dy}{dx} = e^x + x^2$$

$$\Rightarrow \quad e^y = e^x + \frac{x^3}{3} + c$$

15.7 (2)

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = 1 + v + v^2$$

$$\Rightarrow \quad \frac{dv}{1+v^2} = \frac{dx}{x}$$

on integration

$\tan^{-1} v = \ell nx + c$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) = \ell nx + c$$

15.8 (4)

Let $x = X + h$

$y = Y + k$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dY}{dX} = \frac{3Y - 7X + (3k - 7h - 3)}{3X - 7Y + (3h - 7k + 7)}$$

where $3k - 7h - 3 = 0$

and $3h - 7k + 7 = 0$

$\Rightarrow$ $h = 0,\ k = 1$

hence $\frac{dY}{dX} = \frac{3Y - 7X}{3X - 7Y}$

put $Y = vX$

$$\Rightarrow \quad \frac{dY}{dX} = v + X\frac{dv}{dX} = \frac{3v - 7}{3 - 7v}$$

$$\Rightarrow \quad X\frac{dv}{dX} = \frac{7(1 - v^2)}{7v - 3}$$

$$\Rightarrow \quad \int\left(\frac{7v - 3}{1 - v^2}\right)dv = 7\int\frac{dX}{X}$$

$$\Rightarrow \quad \int\left[\frac{2}{1-v} - \frac{5}{1+v}\right]dv = 7\int\frac{dX}{X}$$

$\Rightarrow$ $-2\ell n(1 - v) - 5\ell n(1 + v) = 7\ell nX + c$

$\Rightarrow$ $[X^7(1 - v)^2(1 + v)^5] = c$

$\Rightarrow$ $(X - Y)^2 (X + Y)^5 = c$

$\Rightarrow$ $(y - x - 1)^2 (y + x - 1)^5 = c$

15.9 (1)

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$\text{integrating factor} = e^{\int\frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

$$\Rightarrow \quad x\,.\,e^{\tan^{-1}y} = \int\frac{\tan^{-1}y}{1+y^2}e^{\tan^{-1}y}.dy$$

$$= \tan^{-1}y.\ e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$$

$$= e^{\tan^{-1}y}\left(\tan^{-1}y - 1\right) + c$$

15.10 (3)

$$\frac{dx}{dy}+\frac{2x}{y} = 10\,y^2$$

Integrating factor = $e^{\int \frac{2}{y}dy} = y^2$

$\Rightarrow \quad x.\,y^2 = \int 10\,y^2.y^2\,dy$

$= 2y^5 + c$

15.11 (1)

$$\left(\frac{d^3y}{dx^3}\right)\left(\frac{d^2y}{dx^2}\right)^5 + 4\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^2 = \frac{d^3y}{dx^3}(x^2-1)$$

hence degree = 2, order = 3

15.12 (1)

$y^2 = 4a(x - b)$

$2yy' = 4a \quad \Rightarrow \quad a = \frac{yy'}{2}$

$\Rightarrow \quad y''\,y + (y')^2 = 0$

$\Rightarrow$ order = 2, degree = 1

15.13 (2)

$$\int \frac{dy}{e^y} = \int (e^x + e^{-x})dx$$

$-\,e^{-y} = e^x - e^{-x} + c$

$e^y\,(e^x - e^{-x} + c) = -\,1$

15.14 (2)

$$\frac{dy}{dx} = \frac{x-y}{x+y}$$

$y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \quad v + \frac{xdv}{dx} = \frac{1-v}{1+v}$

$$\frac{x\,dv}{dx} = \frac{1-2v-v^2}{1+v}$$

$$\int \frac{(1+v)dv}{v^2+2v-1} = -\int \frac{dx}{x}$$

$$\frac{1}{2}\ell n(v^2 + 2v - 1) = -\,\ell nx + c$$

$$\frac{1}{2}\ell n(y^2 + 2xy - x^2) = c$$

$\Rightarrow \quad y^2 + 2xy - x^2 = k$

15.15 (4)

$$\frac{xdy}{dx} - y = -x\tan\left(\frac{y}{x}\right)$$

let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\Rightarrow \quad x\left(v + x\frac{dv}{dx}\right) - vx = -x\tan v$

$\Rightarrow \quad x\frac{dv}{dx} = -\tan v \quad \Rightarrow \quad \int\frac{dv}{\tan v} = -\int\frac{dx}{x}$

$\ell n(\sin v) = -\ell n\, x + c$

$\Rightarrow \quad x\sin v = c$

$\Rightarrow \quad v = \sin^{-1}\left(\frac{c}{x}\right)$

$\Rightarrow \quad y = x\sin^{-1}\left(\frac{c}{x}\right)$

15.16 (2)

$$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}$$

for orthogonal trajectory

$$\frac{dx}{dy} = \left(\frac{y}{x}\right)^{1/3}$$

$x^{1/3}\,dx - y^{1/3}\,dy = 0$

$\Rightarrow \quad x^{4/3} - y^{4/3} = c$

15.17 (1)

$$\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1}$$

$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x+1} = \frac{(x+1)^2 - 3}{x+1}$

I.F. $= e^{-\int\frac{1}{x+1}dx} = \frac{1}{x+1}$

$\Rightarrow \quad \frac{y}{x+1} = \int\frac{(x+1)^2 - 3}{(x+1)^2}dx$

$= x + \frac{3}{x+1} + c$

$\because$ curve passes through (2, 0)

$\Rightarrow \quad c = -3$

$\Rightarrow \quad \frac{y}{x+1} = \frac{x^2 - 2x}{x+1}$

$\Rightarrow \quad y = x^2 - 2x$

15.18 (1)

$x = X + h$

$y = Y + k$

15.19 (1)

15.20 (4)

15.18 **(1)**

$x = X + h$

$y = Y + k \quad \Rightarrow \quad h - 2k + 5 = 0$

$\& \quad 2h - k + 4 = 0$

$\Rightarrow \quad h = -1,\ k = 2$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dY}{dX} = \frac{X - 2Y}{-(2X - Y)}$$

put $\quad Y = vX$

$$\Rightarrow \quad \frac{dY}{dX} = v + \frac{Xdv}{dX} = \frac{1-2v}{-(2-v)}$$

$$\Rightarrow \quad \int \frac{dX}{X} = \int \frac{2-v}{-(v^2-1)} dv$$

$$= \int \left[\frac{-1}{2(v-1)} + \frac{3}{2(v+1)}\right] dv$$

$$\Rightarrow \quad \ell nX = -\frac{1}{2}\ell n(v-1) + \frac{3}{2}\ell n(v+1) + c$$

$$\Rightarrow \quad \ell nX = \ell n\left[\frac{(X+Y)^{3/2}}{(Y-X)^{1/2}} X\right] + c$$

$\Rightarrow \quad (X + Y)^3 = \lambda(Y - X)$

$\Rightarrow \quad (x + y - 1)^3 = \lambda\,(x - y + 3)$

15.19 **(1)**

$$\frac{f'(xy)(x\,dy + y\,dx)}{f(xy)} = xdx$$

$$\Rightarrow \quad \frac{d}{dx}\ell n(f(xy)) = \frac{d}{dx}\left(\frac{x^2}{2}\right)$$

$$\Rightarrow \quad \ell n(f(xy)) = \frac{x^2}{2} + c$$

$$\Rightarrow \quad f(xy) = ke^{x^2/2}$$

15.20 **(4)**

$$\int \frac{y''}{y'} dx = \int \frac{-2y'}{1-y} dx$$

$\Rightarrow \quad \ell ny' = 2\ell n(1 - y) + c$

$y' = c_1\,(1 - y)^2$

$$\int \frac{y'}{(1-y)^2} dx = \int c_1 dx$$

$$\frac{1}{1-y} = c_1x + c_2$$

$$y = 1 - \frac{1}{c_1x + c_2}$$

$$y = \frac{c_1x + c_2 - 1}{c_1x + c_2} = \frac{Ax + B}{Ax + C}$$

15.21 (3)

$$x^2\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)-2xy\frac{dy}{dx}+y^2 = 0$$

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2-\frac{2xy\frac{dy}{dx}-y^2}{x^2} = 0$$

$$\Rightarrow \quad \frac{d}{dx}\left(y\frac{dy}{dx}\right)-\frac{d}{dx}\left(\frac{y^2}{x}\right) = 0$$

$$\Rightarrow \quad y\frac{dy}{dx}-\frac{y^2}{x} = c$$

put $y^2 = t$

$$\Rightarrow \quad 2y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{1}{2}\frac{dt}{dx}-\frac{t}{x}=c$$

$$\frac{dt}{dx}-\frac{2t}{x}=c_1$$

$$\text{I.F} = e^{-\int\frac{2}{x}dx} = \frac{1}{x^2}$$

$$\frac{t}{x^2} = \int\frac{c_1}{x^2}dx$$

$$\frac{t}{x^2} = -\frac{c_1}{x}+c_2$$

$$\frac{y^2}{x^2}=\frac{c_2x-c_1}{x}$$

$$y = \pm\sqrt{x(c_2x-c_1)}$$

15.22 (3)

$$(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$$

$$\text{I.F.} = e^{\int\frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

$$\Rightarrow \quad x.e^{\tan^{-1}y} = \int\frac{e^{2\tan^{-1}y}}{1+y^2}dy$$

$$= \frac{1}{2}e^{2\tan^{-1}y}+c$$

$$\Rightarrow \quad 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y}+c'$$

15.23 (3)

$$\frac{xdy - ydx}{x^2 + y^2} = -dx$$

$$\Rightarrow d\left\{\tan^{-1}\left(\frac{y}{x}\right)\right\} = -dx$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = -x + c$$

$$\Rightarrow y = x \tan(c - x)$$

15.24 (3)

$$x = e^{xy}\left(\frac{dy}{dx}\right)$$

$$\log x = xy\left(\frac{dy}{dx}\right)$$

$$y\,dy = \left(\frac{\log x}{x}\right)dx \Rightarrow \frac{y^2}{2} = \frac{(\log x)^2}{2} + c$$

$$\Rightarrow y^2 = (\log_e x)^2 + 2c$$

$$\Rightarrow y = \pm\sqrt{(\log_e x)^2 + 2c}$$

15.25 (3)

Let $x + y = \mu$. then $1 + \frac{dy}{dx} = \frac{d\mu}{dx}$

So, the given differential equation becomes

$$\left(\frac{\mu - 1}{\mu - 2}\right)\left(\frac{d\mu}{dx} - 1\right) = \left(\frac{\mu + 1}{\mu + 2}\right)$$

$$\Rightarrow \frac{d\mu}{dx} - 1 = \left(\frac{\mu + 1}{\mu + 2}\right)\left(\frac{\mu - 2}{\mu - 1}\right)$$

$$\Rightarrow \frac{d\mu}{dx} = \frac{2\mu^2 - 4}{\mu^2 + \mu - 2}$$

$$\Rightarrow \left(\frac{\mu^2 + \mu - 2}{\mu^2 - 2}\right)d\mu = 2dx \Rightarrow \left(1 + \frac{\mu}{\mu^2 - 2}\right)d\mu = 2dx$$

$$\Rightarrow \mu + \frac{1}{2}\log|\mu^2 - 2| = 2x + c$$

$$\Rightarrow 2(y - x) + \log|(x+y)^2 - 2| = 2c$$

when $x = 1$, we have $y = 1$

$\log 2 = 2c$

$$\Rightarrow 2(y - x) + \log\left|\frac{(x+y)^2 + 2}{2}\right| = 0$$

15.26 **(3)**

S_1 : $y^2 = t$

$$2y\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{t}{2}\frac{dt}{dx} + (x + t) = 0$$

this is a homogeneous equation

S_2 : Not always true. consider the equation in S_1

15.27 **(2)**

S_1 : $\text{I.F.} = e^{\int \frac{1}{x\ell nx}dx} = e^{\ell n(\ell nx)} = \ell nx$

S_2 : $y = A \sin(x + B)$

$$\Rightarrow \frac{dy}{dx} = A\cos(x + B)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -A\sin(x + B)$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

15.28 **(1)**

$y = cx^2$

$$\frac{dy}{dx} = 2cx$$

eliminating 'c"

$$y = \left(\frac{1}{2x}\frac{dy}{dx}\right)x^2 \Rightarrow 2y = \frac{xdy}{dx}$$

equation of orthogonal trajectory

$$2y = -x\frac{dx}{dy}$$

$$\Rightarrow y^2 = -\frac{x^2}{2} + c$$

$x^2 + 2y^2 = 2c$

15.29 **(2)**

Slope of ray

slope of nor

15.30 **(1)**

Solving

of the

15.29 **(2)**

Slope of ray = $\frac{y}{x}$

slope of normal at P(x,y) = $-\frac{dx}{dy}$

$$\Rightarrow \frac{\frac{y}{x}+\frac{dx}{dy}}{1-\frac{y}{x}\cdot\frac{dx}{dy}} = -\frac{dx}{dy}$$

$$\Rightarrow \frac{y}{x}\cdot\frac{dy}{dx}+1 = -1 + \frac{y}{x}\cdot\frac{dx}{dy}$$

$$\Rightarrow y\left(\frac{dx}{dy}\right)^2 + 2x\left(\frac{dy}{dx}\right) = y \text{ is a}$$

differentiable equation of the curve which is satisfied by $y^2 = 2x + 1$.

15.30 **(1)**

Solving the equation of the asymptotes the centre is at x = 1 & y = 0. Since e = $\sqrt{2}$, the equation of the family of the rectangular hyperbola is

$$\frac{(x-1)^2}{a^2} - \frac{(y-0)^2}{a^2} = 1$$

$$\Rightarrow 2(x-1) - 2y\frac{dy}{dx} = 0$$

$$\Rightarrow (x-1) - y\frac{dy}{dx} = 0$$

Hence, (1) is the correct answer.

16. QUADRATIC EQUATION

16.1 **(1)**

α, β are roots of equation $x^2 + px + q = 0$ so

$\alpha + \beta = -p$

$\alpha\beta = q$

and $\alpha^2 + p\alpha + q = 0$

$\alpha + p = \dfrac{-q}{\alpha}$

so $(\alpha + p)^{-2} = \dfrac{\alpha^2}{q^2}$ similarly $(\beta + p)^{-2} = \dfrac{\beta^2}{q^2}$

so required equation

$$x^2 - \left(\frac{\alpha^2+\beta^2}{q^2}\right)x + \frac{\alpha^2\beta^2}{q^4} = 0 \Rightarrow x^2 - \left(\frac{p^2-2q}{q^2}\right)x + \frac{1}{q^2} = 0 \Rightarrow q^2x^2 - (p^2 - 2q)\,x + 1 = 0$$

16.2 **(1)**

One root of equation $x^2 + Ax + 12 = 0$ is 4.

sum of roots $= -A$

product of roots $= 12$

so other root is 3.

$A = -7$

Now roots of equation $x^2 + 2Ax + B = 0$ are equal

so $4A^2 - 4B = 0$

$B = A^2 \Rightarrow B = 49$

16.3 **(3)**

roots are real so $4q^2 - 4pr \geq 0$

and $4pr - 4q^2 \geq 0$

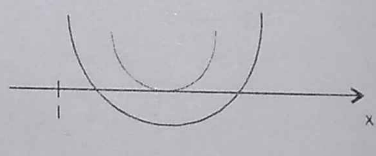
both inequations are true if

$q^2 - pr = 0 \Rightarrow \dfrac{p}{q} = \dfrac{q}{r}$

16.4 **(4)**

$f(x) = 2x^2 + \lambda x - (\lambda + 1) = 0$

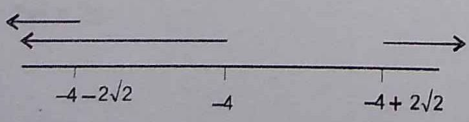
$D = \lambda^2 + 8(\lambda + 1) \geq 0$



$\lambda \in [-\infty, -4 - 2\sqrt{2}] \cup [-4 + 2\sqrt{2}, \infty]$ ....(1)

$\dfrac{-b}{2a} = \dfrac{-\lambda}{4} > 1 \Rightarrow \lambda < -4$ ....(2)

$f(1) = 2 + \lambda - (\lambda + 1) > 0, \ \forall \ \lambda \in R$



$\Rightarrow \lambda \in (-\infty, -4 - 2\sqrt{2}]$

16.5 (1)
$(p^2 + p - 1)x^2 - (2p^2 - 4)x - (3p^2 + 9p + 3) = 0$
$\Rightarrow \quad (x^2 - 2x - 3)p^2 + (x^2 - 9)p - (x^2 - 4x + 3) = 0$
It is identity in p so
$x^2 - 2x - 3 = 0 \quad \Rightarrow \quad x = 3, -1$
$x^2 - 9 = 0 \quad \Rightarrow \quad x = \pm 3$
$x^2 - 4x + 3 = 0 \quad \Rightarrow \quad x = 3, 1$
so for identity x = 3

16.6 (4)
It is identity in x so
$r^2 - 2r + 1 = 0 \quad \Rightarrow \quad r = 1$
$r^2 - 3r + 2 = 0 \quad \Rightarrow \quad r = 1, 2$
$r^2 + 4r + 3 = 0 \quad \Rightarrow \quad r = -1, -3$
No common value of r is possible so it is not possible.

16.7 (2)
$2x^2 - 5x - 7 = 0$ roots are α, β

so $\alpha + \beta = \frac{5}{2}$

$\alpha\beta = -\frac{7}{2}$

Now roots are $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

$$\text{Sum} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\frac{25}{4} + 7}{\frac{-7}{2}} = -\frac{53}{14}$$

product = 1

so equation is $x^2 + \frac{53}{14}x + 1 = 0$

$14x^2 + 53x + 14 = 0$

16.8 (3)
$(\ell - m)x^2 - 5(\ell + m)x - 2(\ell - m) = 0$
$D = 25(\ell + m)^2 - 4(\ell - m)[-2(\ell - m)]$
$= 25(\ell^2 + m^2 + 2\ell m) + 8(\ell^2 + m^2 - 2\ell m) = 33(\ell^2 + m^2) + 34\ell m > 0$
so roots are real unequal.

16.9 (2)
$ax^2 + bx + c = 0$ roots are imaginary so $b^2 - 4ac < 0$
$(4c + 2b + a)x^2 - 2(a + b)x + a = 0$
$D = 4(a + b)^2 - 4a(4c + 2b + a)$
$= 4[a^2 + b^2 + 2ab - 4ac - 2ab - a^2]$
$= 4(b^2 - 4ac) < 0$
so roots are imaginary.

16.10 (3)
$x^2 + 2ax + 2a^2 = 0$
$D = 4a^2 - 8a^2 = -4a^2 > 0$ (a is purely imaginary)

roots $x = \frac{-2a \pm \sqrt{D}}{2}$

so roots are imaginary.

16. (2)

$x^2 + 3x + 4 = 0$

$D = 9 - 16 < 0$ so roots are imaginary

so both roots of given equation are common.

$\frac{2}{1} = \frac{a}{3} = \frac{8}{4} \quad \Rightarrow \quad a = 6$

16.12 (2)

Graph is upward parabola so $a > 0$

$c > 0$

x - co-ordinate of vertex $\frac{-b}{2a} < 0 \quad \Rightarrow \quad -b < 0$

$b > 0$

$f(-2) = 4a - 2b + c < 0$

$f(2) = 4a + 2b + c > 0$

16.13 (2)

$x^2 + x + 3 < x^2 - 4x + 4 - 6$ and $x^2 - 2 < x^2 + x + 3$

$5x < -5$ and $x > -5$

$x < -1$

so $x \in (-5, -1)$

16.14 (3)

Roots are of opposite sign so

$f(0) < 0$

$\lambda^2 - 5\lambda + 6 < 0$

$(\lambda - 2)(\lambda - 3) < 0$

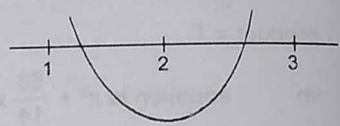
$\Rightarrow \quad \lambda \in (2, 3)$

16.15 (4)

$f(1) > 0$

$f(2) < 0$

$f(3) > 0$



so $1 + \lambda + 1 + \lambda^2 - 3\lambda - 6 > 0 \Rightarrow \lambda^2 - 2\lambda - 4 > 0$

$\lambda \in (-\infty, 1-\sqrt{5}) \cup (1+\sqrt{5}, \infty)$

$f(2) = 4 + 2\lambda + 2 + \lambda^2 - 3\lambda - 6 < 0 \quad \Rightarrow \quad \lambda^2 - \lambda < 0$

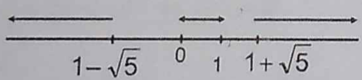
$\lambda(\lambda - 1) < 0$

$\lambda \in (0, 1)$

$f(3) = 9 + 3\lambda + 3 + \lambda^2 - 3\lambda - 6 > 0$

$\lambda^2 + 6 > 0$

$\lambda \in R$



$\lambda \in \phi$

16.16 (1)

$2x^2 + y^2 - 2xy - 4y + 8 = 0 \Rightarrow 4x^2 + 2y^2 - 4xy - 8y + 16 = 0$

$(2x - y)^2 + (y - 4)^2 = 0 \quad \Rightarrow 2x - y = 0$ and $y = 4$ $(x, y) \equiv (2, 4)$

16.17 (3)

Using factor theorem

$\lambda x^3 + x^3 + \mu x^3 - x^3 = 0$

$\Rightarrow \quad \lambda + \mu = 0 \quad ...... (1)$

$\lambda x^3 - 2x^3 + 4\mu x^3 + 8x^3 = 0$

$\lambda + 4\mu + 6 = 0 \quad ...... (2)$

by solving (1) & (2)

$\lambda = 2$

$\mu = -2$

16.18 **(4)**

$||x-2|-3| < 4$

$-4 < |x-2|-3 < 4$

$-1 < |x-2| < 4$

$\Rightarrow \quad |x-2| < 4$

$-4 < x-2 < 4$

$-2 < x < 6$

16.19 **(3)**

$x^2 - 2x = t \geq -1$

$t^2 - 3t + (k+2) = 0$

for two real solution

case (i) : $f(-1) < 0$

$1 + 3 + k + 2 < 0$

$k < -6$

case (ii) : $D = 0$

$\Rightarrow 9 - 4(k+2) = 0$

$k = \frac{1}{4}$

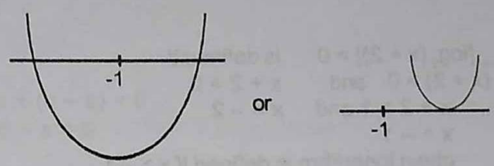
$f(-1) > 0$

$\Rightarrow k > -6$

so $k = \frac{1}{4}$

from case (i) & case (ii)

$k \in (-\infty, -6) \cup \left\{\frac{1}{4}\right\}$



16.20 **(3)**

$(\alpha-\gamma)(\alpha-\delta) = \alpha^2 - (\gamma+\delta)\alpha + \gamma\delta$

$= \alpha^2 + p\alpha - 4$ (because $\gamma + \delta = -p$, $\gamma\delta = -4$)

α is root of $x^2 + px + 7 = 0$

$\Rightarrow \quad \alpha^2 + p\alpha + 7 = 0 \quad \Rightarrow \quad \alpha^2 + p\alpha = -7$

Now $\quad (\alpha-\gamma)(\alpha-\delta) = \alpha^2 + p\alpha - 4 = -7 - 4 = -7 - 4 = -11$

16.21 **(1)**

$x^2 - 6x + 4 = 0$ roots are α, β

so $\quad \alpha^2 - 6\alpha + 4 = 0 \quad \Rightarrow \quad \alpha - 6 = -\frac{4}{\alpha}$

similarly $\quad \beta - 6 = -\frac{4}{\beta}$

Now $\quad (\alpha-6)^{-2} + (\beta-6)^{-2} = \frac{\alpha^2}{16} + \frac{\beta^2}{16} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{16} = \frac{36 - 2\times 4}{16} = \frac{28}{16} = \frac{7}{4}$

16.22 **(3)**

$2x^2 + 9x + 4a = 0$ one root is α then 2α is root at $2x^2 + 3x + a = 0$

so $\quad 2\alpha^2 + 9\alpha + 4a = 0 \quad$ .... (1)

$8\alpha^2 + 6\alpha + a = 0 \quad$ .... (2)

by solving (1) and (2) $\alpha = -\frac{a}{2}$

from (1)

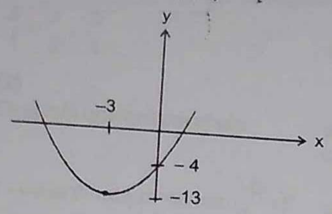
$2.\ \frac{a^2}{4} - \frac{9a}{2} + 4a = 0 \quad$ so $a = 1$

16.23 **(4)**

$y(-4) = 16 - 24 - 4 = -12$

$y(3) = 9 + 18 - 4 = 23$

so range is $[-13, 23)$



16.24 **(4)**

$\log_{1/4}[\log_2(x+2)] > 0$ is defined if

$\log_2(x+2) > 0$ and $x + 2 > 0$

$\Rightarrow$ $x + 2 > 1$ and $x > -2$

$\Rightarrow$ $x > -1$

given logarithm is defined if $x > -1$

Now $\log_{1/4}[\log_2(x+2)] > 0$

$\Rightarrow$ $\log_2(x+2) < 1$

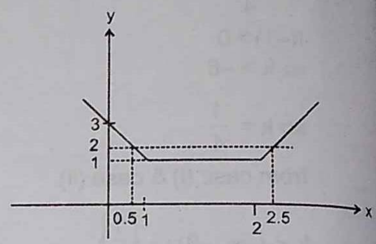
$x + 2 < 2$

$x < 0$

so $x \in (-1, 0)$ .....(1)

$|x-1| + |x-2| < 2$

so $x \in \left(\frac{1}{2}, \frac{5}{2}\right)$ .....(2)



Intersection of (1) and (2) is empty set.

16.25 **(1)**

Roots are α, β, γ

so $\alpha + \beta + \gamma = 9$

$\alpha\beta\gamma = 27$

$\frac{\alpha+\beta+\gamma}{3} = (\alpha\beta\gamma)^{1/3} = 3$

so AM = GM

$\Rightarrow$ $\alpha = \beta = \gamma = 3$

$P = \alpha\beta + \beta\gamma + \alpha\gamma = 27$

16.26 **(4)**

α, β, γ are root of $x^3 - 5x^2 + x - 2 = 0$

Now $y = \frac{\alpha+2}{\alpha-2}$ $\Rightarrow$ $\alpha = \frac{2(y+1)}{y-1}$

it is root of given equation so

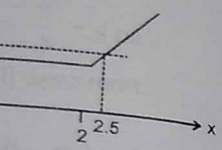
$$\frac{8(y+1)^3}{(y-1)^3} - 5.4\frac{(y+1)^2}{(y-1)^2} + \frac{2(y+1)}{(y-1)} - 2 = 0$$

$8(y+1)^3 - 20(y+1)^2(y-1) + 2(y+1)(y-1)^2 - 2(y-1)^3 = 0 \Rightarrow 3y^3 - 2y^2 - 9y - 8 = 0$

roots of this equation are $\frac{\alpha+2}{\alpha-2}, \frac{\beta+2}{\beta-2}, \frac{\gamma+2}{\gamma-2}$

so product of roots

$$\left(\frac{\alpha+2}{\alpha-2}\right)\left(\frac{\beta+2}{\beta-2}\right)\left(\frac{\gamma+2}{\gamma-2}\right) = \frac{8}{3}$$



16.27 **(2)**

$x^2 - 2x = t \geq -1$

$t^2 - 3t + (k + 2) = 0$

for four real solution t must lies in $(-1, \infty)$

$D = 9 - 4(k + 2) > 0$

$9 - 4k - 8 > 0$

$4k < 1 \quad \Rightarrow \quad k < \frac{1}{4}$

$f(-1) > 0$

$1 + 3 + k + 2 > 0$

$k > -6$

so $\quad k \in \left(-6, \frac{1}{4}\right)$

16.28 **(4)**

$\alpha^2 + p\alpha + q = 0 \quad \Rightarrow \quad (p + r)\,\alpha + (q - s) = 0$

$\alpha^2 - r\alpha + s = 0 \quad \Rightarrow \quad p + r + q - s = 0$

16.29 **(3)**

S-1 Let $f(x) = (x - p)(x - r) + \lambda\,(x - q)(x - s)$

$f(p) = \lambda(p - q)\,(p - s)$

$f(q) = (q - p)\,(q - r)$

$f(r) = \lambda(r - q)\,(r - s)$

$f(s) = (s - p)\,(s - r)$

If $\lambda > 0$ then $f(p) > 0$, $f(q) < 0$, $f(r) < 0$, $f(s) > 0$

$\Rightarrow$ $f(x) = 0$ has one real root between p and q and other real root between r and s.

S-2 obviously false

16.30 **(2)**

S-1 $x^2 - bx + c = 0$, α, β are roots such that

$|\alpha - \beta| = 1$

$\Rightarrow$ $(\alpha + \beta)^2 - 4\,\alpha\,\beta = 1$

$b^2 - 4c = 1$

S-2 Equation $4abc\,x^2 + (b^2 - 4ac)x - b = 0$

$D = (b^2 - 4ac)^2 + 4ab^2c$

$D = (b^2 + 4ac)^2 > 0$ roots are real and unequal.

16.31 **(4)**

S-1 For identity in x

$a^2 - 3a + 2 = 0 \quad \Rightarrow \quad a = 1, 2$

$a^2 - 5a + 6 = 0 \quad \Rightarrow \quad a = 2, 3$

$a^2 - 4 = 0 \quad \Rightarrow \quad a = \pm 2$

so $\quad a = 2$

16.32 **(4)**

If a, b, c are real then

roots of equation $ax^2 + bx + c = 0$ are non-real if $b^2 - 4ac < 0$

17. SEQUENCE & SERIES

17.1 **(3)**

$\because$ 2p, q, 2r are in G.P.

$\Rightarrow q^2 = 4pr \Rightarrow$ roots of $px^2 + qx + r = 0$ are equal

$\Rightarrow \alpha^2 = 4\alpha - 4 \Rightarrow (\alpha - 2)^2 = 0 \Rightarrow \alpha = 2$

17.2 **(2)**

Let d be the common difference of A.P.

So $\quad \frac{m}{2}[2a + (m-1)d] = 0$

$\Rightarrow \quad d = \frac{-2a}{m-1}$

Now $\quad S_{m+n} = s_m$ + sum of next n terme

$\Rightarrow \quad$ Sum of next n terms = S_{m+n}

$= \frac{m+n}{2}[2a + (m+n-1)d]$

Put the value of d

sum of next n terms = $\frac{-an(m+n)}{m-1}$

17.3 **(1)**

Discriminant of first equation $4b^2 - 4ac = 0$ (because $b^2 = ac$)

So fisrt equation has equal roots, Let roots be α

then $\quad \alpha + \alpha = \frac{-2b}{a} \Rightarrow \alpha = -b/a$

Given two equation have a comon root

So $\quad d(-b/a)^2 + 2e(-b/a) + f = 0$

$\Rightarrow \quad d\,b^2 - 2eba + f\,a^2 = 0$

$\Rightarrow \quad \frac{d}{a} - \frac{2e}{b} + \frac{fa}{b^2} = 0$ but $b^2 = ac$

So $\quad \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$

$\Rightarrow \quad \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are is A.P.

17.4 **(4)**

When $\quad d = -2$, sum $= -5$

So $-5 = \frac{5}{2}(2a + 4.(-2))$

$\Rightarrow a = 3$

So actual sum when d = 2 is = $\frac{5}{2}[2 \times 3 + 4 \times 2] = 35$

17.5 (4)

$L_1\ L_3\ L_5$ $\quad$ $L_2\ L_4\ L_6$

$E_1\ E_3\ E_5$ $\quad$ $E_2\ E_4\ E_6$

Number of ways = $3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right).3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right) = 4$

17.6 (4)

$t_m = t_n$

$1 + (m-1).10 = 31 + (n-1).5$

$10m - 9 = 5n + 26$

$10m = 5n + 35 = 5(n+7)$

$m = \frac{n+7}{2} = \lambda$

$m = \lambda$ and $n = 2\lambda - 7$

$m \le 100$ and $n \le 100$

$\lambda \le 100$ and $2\lambda - 7 \le 100 \Rightarrow \lambda \le 53\frac{1}{2}$

so $1 \le \lambda \le 53$ [No. of common term = 53]

$t_{53} = 1 + 52.10 = 521$

17.7 (2)

$$\frac{S_{3r}-S_{r-1}}{S_{2r}-S_{2r-1}} = \frac{\frac{3r}{2}[2a+(3r-1)d]-\frac{(r-1)}{2}[2a+(r-2)d]}{\frac{2r}{2}[2a+(2r-1)d]-\frac{(2r-1)}{2}[2a+(2r-2)d]}$$

$$= \frac{2a(2r+1)+d(8r^2-2)}{2a+d(4r-2)} = (2r+1)\left[\frac{2a+2(2r-1)d}{2a+2(2r-1)d}\right] = 2r+1$$

17.8 (2)

$t_r = 2^{\frac{r}{2}} + 2^{\frac{-r}{2}}$.

$t_r^2 = 2^r + 2^{-r} + 2$

$$\sum_{r=1}^{10} t_r^2 = (2 + 2^2 + ... + 2^{10}) + \left(\frac{1}{2}+\frac{1}{2^2}+......+\frac{1}{2^{10}}\right) + 20$$

$$= 2(2^{10}-1) + \left(1-\frac{1}{2^{10}}\right) + 20 = 2^{11} - \frac{1}{2^{10}} + 19 \Rightarrow \frac{2^{21}-1}{2^{10}} + 19$$

17.9 (1)

$(1+x)(1+x^2)...........(1+x^{128}) = 1 + x + x^2 + ...+x^n$

$\frac{(1-x)}{(1-x)}[(1+x)(1+x^2).......(1+x^{128})] = \frac{1-x^{n+1}}{1-x}$

$(1-x^2)(1+x^2)(1+x^4).....(1+x^{128}) = 1 - x^{n+1}$

$(1-x^4)(1+x^4)(1+x^8) + ...(1+x^{128}) = 1 - x^{n+1}$

$(1-x^{128})(1+x^{128}) = 1 - x^{n+1}$

$\Rightarrow 1 - x^{256} = 1 - x^{n+1}$

so $n + 1 = 256 \Rightarrow n = 255$

17.10 (1)

Let the first term of A.P. be a and common difference be d

$T_{20^{th}}$ of H.P. = 1 $\qquad$ $T_{20^{th}}$ of A.P. = 1

$a + 19d = 1$

$T_{30^{th}}$ of H.P. = $\frac{-1}{17}$ $\qquad$ $t_{30^{th}}$ of A.P. = –17

$a + 29d = -17$

$\Rightarrow \quad a = \frac{176}{5}, \; d = \frac{-9}{5}$

$T_{n^{th}}$ term of H.P. = $\frac{1}{a+(n-1)d} = \frac{1}{\frac{176}{5}+(n-1)\left(\frac{-9}{5}\right)} = \frac{5}{185-9n}$

For the largest term 185 – 9n will be least n = 20 the 20^{th} term of the given H.P. is the largest term

$T_{20} = \frac{5}{185-180} = 1$

17.11 (2)

Let the numbers be a and b

$3A + G^2 = 36$ and $G^2 = AH$

$\Rightarrow AH + 3A = 36 \Rightarrow 3A + \frac{21}{5}A = 36 \Rightarrow A = 5$

$\frac{a+b}{2} = 5 \Rightarrow a + b = 10$

and $\frac{2ab}{a+b} = \frac{21}{5} \Rightarrow ab = 21$

$a^2 + b^2 = (a + b)^2 - 2ab$

$= 10^2 - 42 \Rightarrow 100 - 42 = 58$

17.12 (2)

$28, A_1, A_2 .... A_{10}, A_{11}, 10$

$d = \frac{10-28}{11+1} = \frac{-18}{12} = \frac{-3}{2}$

$A_1 = 28 + d \Rightarrow$ clearly not integral

The A.M.'s will be integral only when 2d, 4d....... types of terms occur

So, $A_2, A_4, A_6, A_8, A_{10}$ are the A.M.

Total number = 5

17.13 (1)

$2, \; G_1, G_2, G_3, G_4, \; 486$

$r = \left(\frac{486}{2}\right)^{\frac{1}{4+1}} = (243)^{1/5} = 3$

$G_1 = ar = 2(3) = 6$

$G_2 = ar^2 = 2.(9) = 18$

$G_3 = ar^3 = 2.27 = 54$

$G_4 = ar^4 = 2.81 = 162$

$G_1 + G_2 + G_3 + G_4 = 162 + 54 + 18 + 6 = 240$

17.14 **(1)**

First find out 4 AM between the two numbers $\frac{3}{2}$ $A_1 A_2 A_3 A_4$ $\frac{13}{2}$

$d = \frac{\frac{13}{2} - \frac{3}{2}}{4+1} = 1$ $\qquad$ $d = 1$

$A_1 = \frac{3}{2} + 1 = \frac{5}{2}$ $\qquad$ $A_2 = \frac{3}{2} + 2 = \frac{7}{2}$

$A_3 = \frac{3}{2} + 3 = \frac{9}{2}$ $\qquad$ $A_4 = \frac{3}{2} + 4 = \frac{11}{2}$

$H_1 = \frac{1}{A_1} = \frac{2}{5}$ $\qquad$ $H_2 = \frac{1}{A_2} = \frac{2}{7}$

$H_3 = \frac{1}{A_3} = \frac{2}{9}$ $\qquad$ $H_4 = \frac{1}{A_4} = \frac{2}{11}$

$H_1.H_2.H_3.H_4 = \frac{2^4}{5.7.9.11} = \frac{16}{5.7.9.11}$

17.15 **(3)**

Given line 3x + 4y = 5

x^2y^3 can be obtained from this $\left(\frac{3}{2}x\right)^2\left(\frac{4}{3}y\right)^3$

3x can be break into two equal part and 4y can be break into three equal part

$$\frac{\frac{3x}{2} + \frac{3x}{2} + \frac{4y}{3} + \frac{4y}{3} + \frac{4y}{3}}{5} \geq \left[\left(\frac{3x}{2}\right)^2\left(\frac{4y}{3}\right)^3\right]^{1/5}$$

$1 \geq \frac{9.64}{4.27}.x^2y^3$

$x^2y^3 \leq \frac{4.27}{9.64} \leq \frac{3}{16}$

maximum value is $\frac{3}{16}$

17.16 **(3)**

$$\frac{4^{\sin^2 x} + 4^{\cos^2 x}}{2} \geq \left(4^{\sin^2 x}.4^{\cos^2 x}\right)^{1/2}$$

$4^{\sin^2 x} + 4^{\cos^2 x} \geq 4$

Min value = 4

17.17 **(1)**

A.M. ≥ H.M.

b is the harmonic mean of a and c $\frac{a+c}{2} \geq b$

c is the harmonic mean of b and d $\frac{b+d}{2} \geq c$

Now

a + c ≥ 2b

b + d ≥ 2c

Add the two

a + c + b + d ≥ 2(b + c)

a + d ≥ b + c

17.18 (2)

A.M. ≥ H.M.

$$\frac{a+b+c}{3} \geq \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$$

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$$

17.19 (1)

$S = 2 + 5 + 10 + 17 + 26 + ... + t_n$

$S = \quad 2 + 5 + 10 + 17 + ... + t_{n-1} + t_n$

Subtract

$0 = 2 + [3 + 5 + 7 + ... + (n-1) \text{ terms}] - t_n$

$t_n = 2 + [3 + 5 + 7 + ... + (n-1) \text{ terms}]$

$$t_n = 2 + \frac{n-1}{2}[2.3 + (n-2).2]$$

$$T_n = 2 + \frac{(n-1)}{2}.2(n+1) = 2 + n^2 - 1$$

$T_n = n^2 + 1$

$$S_n = \sum T_n = \sum n^2 + \sum 1 = \frac{n(n+1)(2n+1)}{6} + n = n\left[\frac{2n^2+3n+7}{6}\right] = \frac{n}{6}(2n^2 + 3n + 7)$$

17.20 (2)

$S = 1 + 4 + 13 + 40 + 121 + ... + t_n$

$S = \quad 1 + 4 + 13 + 40 + ... + t_{n-1} + t_n$

$0 = 1 + [3 + 9 + 27 + 81 + ... (n-1) \text{ terms}] - t_n$

$$t_n = 1 + 3\left[\frac{3^{n-1}-1}{3-1}\right] \Rightarrow \frac{3^n-3}{2}+1 = \frac{3^n-1}{2}$$

$$t_n = \frac{3^n-1}{2}$$

$$S_n = \sum t_n = \frac{1}{2}[3+3^2+......+3^n] - \frac{n}{2}$$

$$S = \frac{1}{2}\left[\frac{3(3^n-1)}{2} - n\right] = \frac{1}{4}[3^{n+1} - 2n - 3]$$

$$S = \frac{1}{4}[3^{n+1} - 2n - 3]$$

17.21 (3)

$T_r = [1 + (r-1).1][n + (r-1).(-1)]$

$= r[n - r + 1]$

$T_r = (n+1)r - r^2$

$$S_n = \sum_{r=1}^{n} T_r = (n+1)\sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2 = \frac{(n+1)n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{2}\left[(n+1) - \frac{2n+1}{3}\right] = \frac{n(n+1)(n+2)}{6}$$

17.22 (1)

$$T_r = \frac{1}{[1+(r-1).2]\,[3+(r-1).2]} = \frac{1}{(2r-1)(2r+1)} = \frac{1}{2}\left[\frac{(2r+1)-(2r-1)}{(2r-1)(2r+1)}\right]$$

$$T_r = \frac{1}{2}\left[\frac{1}{2r-1} - \frac{1}{2r+1}\right] \Rightarrow S_n = \sum_{r=1}^{n} T_r \Rightarrow S_n = \frac{1}{2}\left[1 - \frac{1}{2n+1}\right] = \frac{n}{2n+1}$$

17.23 (1)

$S_\infty = 1 - 3x + 5x^2 - 7x^3 + \ldots\ldots \infty$

Here A.P. is

1, 3, 5, 7. ....

G.P. 1, $-x$, x^2, $-x^3$..........

Common ratio is $-x$

$$S = 1 - 3x + 5x^2 - 7x^3 + \ldots\ldots\infty$$

$$-xS = \quad -x + 3x^2 - 5x^3 + \ldots\ldots\infty$$

$$(1 + x)S = 1 - 2x + 2x^2 - 2x^3 + \ldots\ldots \infty$$

$$(1 + x)S = 1 - [2x - 2x^2 + 2x^3 + \ldots\ldots\infty]$$

$$(1 + x)S = 1 - \frac{2x}{1+x}$$

$$S = \frac{1-x}{(1+x)^2}$$

17.24 (1)

$$S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots\ldots + \frac{(3n-2)}{5^{n-1}}$$

$$\frac{1}{5}S = \quad \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \ldots\ldots + \frac{(3n-5)}{5^{n-1}} + \frac{(3n-2)}{5^n}$$

Now subtract the two

$$\frac{4}{5}S = 1 + \left[\frac{3}{5} + \frac{3}{5^2} + \ldots\ldots\ldots\ldots(n-1) \text{ terms पद}\right] - \frac{(3n-2)}{5^n}$$

$$\frac{4}{5}S = 1 + \frac{\left(\frac{3}{5}\right)\left[1-\left(\frac{1}{5}\right)^{n-1}\right]}{\left[1-\frac{1}{5}\right]} - \frac{(3n-2)}{5^n}$$

$$\frac{4}{5}S = 1 + \frac{3}{4}\left[1 - \frac{1}{5^{n-1}}\right] - \frac{(3n-2)}{5^n}$$

$$S = \frac{5}{4} + \frac{15}{16}\left[1 - \frac{1}{5^{n-1}}\right] - \frac{(3n-2)}{4.5^{n-1}}$$

17.25 (2)

$$T_r = \frac{r}{(r+1)!} = \frac{r+1-1}{(r+1)!} \Rightarrow T_r = \frac{1}{r!} - \frac{1}{(r+1)!} \Rightarrow \sum_{r=1}^{n} T_r = \sum_{r=1}^{n}\left(\frac{1}{r!} - \frac{1}{(r+1)!}\right)$$

$$= \frac{1}{1!} - \frac{1}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

17.26 **(3)**

Let a = a, b = ar, c = ar^2

as a, b, c are in G.P.

Now a – b, c – a, b – c → H.P.

$$\frac{1}{a-b}, \frac{1}{c-a}, \frac{1}{b-c} \rightarrow AP$$

$$\frac{2}{ar^2-a} = \frac{1}{a-ar} + \frac{1}{ar-ar^2} \Rightarrow \frac{2}{r^2-1} = \frac{1}{1-r} + \frac{1}{r(1-r)} = \frac{1+r}{r(1-r)}$$

$$\frac{-2}{(1-r)(1+r)} = \frac{(1+r)}{r(1-r)}$$

$r^2 + 2r + 1 + 2r = 0$

$r^2 + 4r + 1 = 0$

Now a + 4b + c ⇒ a + 4ar + ar^2 = $a(r^2 + 4r + 1) = 0$

17.27 **(2)**

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}$$

$$S_n = (2-1) + \left(2-\frac{1}{2}\right) + \left(2-\frac{1}{3}\right) + \ldots\ldots + \left(2-\frac{1}{50}\right)$$

$$S_n = 2.50 - \left[1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots+\frac{1}{50}\right]$$

$S_n = 100 - H_{50}$

17.28 **(1)**

A.M. ≥ H.M.

$$\frac{b+c}{2} \geq \frac{2bc}{b+c} \Rightarrow \frac{bc}{b+c} \leq \frac{1}{4}(b+c) \quad \ldots(1)$$

Similarly

$$\frac{ac}{a+c} \leq \frac{1}{4}(a+c) \quad \ldots(2)$$

$$\frac{ab}{a+b} \leq \frac{1}{4}(a+b) \quad \ldots(3)$$

add all these

$$\frac{bc}{b+c} + \frac{ac}{a+c} + \frac{ab}{a+b} \leq \frac{1}{4}2(a+b+c) \Rightarrow \frac{bc}{b+c} + \frac{ac}{a+c} + \frac{ab}{a+b} \leq \frac{1}{2}(a+b+c)$$

17.29 **(1)**

Given $S_n = \sum_{r=1}^{n} I(r) = n(2n^2 + 9n + 13)$

$S_r = r(2r^2 + 9r + 13)$

$S_{r-1} = (r-1)[2(r-1)^2 + 9(r-1) + 13]$

$I(r) = S_r - S_{r-1}$

$I(r) = 6r^2 + 12r + 6 = 6(r+1)^2$

$$\sqrt{I(r)} = \sqrt{6}(r+1) \Rightarrow \sum_{r=1}^{n}\sqrt{I(r)} = \sqrt{6}\sum_{r=1}^{n}(r+1)$$

$$= \sqrt{6}\sum_{r=1}^{n} r + \sum_{r=1}^{n}.1 \Rightarrow \sqrt{6}\left[\frac{n(n+1)}{2}+n\right] \Rightarrow \frac{\sqrt{6}}{2}\left[n^2+3n\right] \Rightarrow \sqrt{\frac{3}{2}}(n^2+3n)$$

17.30 (1)

$$\left(\frac{ac+ab-bc}{abc}\right)\left(\frac{ab+bc-ac}{abc}\right) \Rightarrow \left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$$

Now $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$$\left[\frac{1}{2a}+\frac{1}{2c}+\frac{1}{c}-\frac{1}{a}\right]\left[\frac{1}{c}+\frac{1}{a}-\frac{1}{2a}-\frac{1}{2c}\right] \Rightarrow \left[\frac{3}{2c}-\frac{1}{2a}\right]\left[\frac{1}{2a}+\frac{1}{2c}\right] = \frac{(3a-c)(a+c)}{4a^2c^2}$$

17.31 (1)

$$\frac{1}{a_1^2-a_2^2} + \frac{1}{a_2^2-a_3^2} + \ldots\ldots + \frac{1}{a_{n-1}^2-a_n^2}$$

$$= \sum_{k=2}^{n}\frac{1}{a_{k-1}^2-a_k^2} = \sum_{k=2}^{n}\frac{1}{a^2r^{2k-4}(1-r^2)}$$

$$= \frac{1}{a^2(1-r^2)}\cdot\sum_{k=2}^{n}\frac{1}{(r^{2k-4})} = \frac{1}{a^2(1-r^2)}\cdot\frac{(1/r^2)^{n-1}-1}{(1/r^2-1)} = \frac{r^2(1-r^{2n-2})}{a^2r^{2n-2}(1-r^2)^2}$$

17.32 (3)

For two positive numbers $(GM)^2 = (AM)(HM)$

This is not true for numbers greater than 2. Hence statement-1 is true but statement-2 is false.

17.33 (3)

$2b = a + c$

$c^2 = ab$

To prove that : c, a, b are in HP

$$a = \frac{2bc}{b+c} = \frac{c(a+c)}{b+c} \Rightarrow \frac{ac+c^2}{b+c} \Rightarrow \frac{ac+ba}{b+c}$$

$= a$

Hence c, a, b are in HP

(2) Eliminating a from the two equation

$\frac{c^2}{b} + c = 2b \Rightarrow c^2 + bc = 2b^2$

$\Rightarrow 2b^2 - bc - c^2 = 0 \quad \Rightarrow 2b^2 - 2bc + bc - c^2 = 0 \quad \Rightarrow 2b(b - c) + c(b - c) = 0$

$c = -2b$ or $b = c$ which is not possible

$2b = a + c$

$4b = a$

$a : b : c = 4b : b : -2b = 4 : 1 : -2$

17.34 (1)

Let

$$S = 1 + 3y + 5y^2 + 7y^3 + \ldots\ldots\infty$$
$$yS = \quad y + 3y^2 + 5y^3 + \ldots\ldots\infty$$

$$(1 - y)S = 1 + 2y + 2y^2 + 2y^3 + \ldots\ldots\infty$$

$$(1-y)S = 1 + \frac{2y}{1-y} = \frac{1+y}{1-y} \Rightarrow S = \frac{(1+y)}{(1-y)^2}$$

Putting $y = 1 - \frac{1}{x} \Rightarrow \quad S = \frac{1+1-\frac{1}{x}}{\left(\frac{1}{x}\right)^2} = 2x^2 - x$

18. Binomial theorem

18.1 (3)

put n = 1

we get $11^{1+2} + 12^{2\times1+1} = 11^3 + 12^3 = 3059$, which is divisible by 133

18.2 (1)

Sol. $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^{42}$ .... + $^{21}C_{21}.\ 2^{21}$

$T_{r+1} = {}^{42}C_r \left(\sqrt{x}\right)^{42-r} . \left(\frac{1}{\sqrt{x}}\right)^r$

x^{21-r} r = 21 (for independent of x)

18.3 (2)

Sol. $\sum_{r=0}^{n} {}^nC_r\ a^r .\ b^{n-r} . \cos(rB - (n-r)A)$

$= \text{Real part of} \left(\sum_{r=0}^{n} {}^nC_r\ a^r .\ b^{n-r}\ e^{i(rB-(n-r)A)}\right)$

$= \text{Real part of} \left[\sum_{r=0}^{n} {}^nC_r\ (a.e^{iB})^r\ (b\ e^{-iA})^{n-r}\right]$

$= \text{Real part of } (ae^{iB} + be^{-iA})^n$

$= \text{Real part of } (a\cos B + i\, a \sin B + b \cos A - i\, b \sin A)^n$

$= (a \cos B + b \cos A)^n = c^n$

18.4 (1)

Sol. ${}^nC_1 + {}^nC_2 = 36$

$n + \frac{n(n-1)}{2} = 36$

$n^2 + n - 72 = 0$

$n = -9, 8$

$\therefore\ n = 8$

$$\frac{T_3}{T_2} = \frac{{}^8C_2(2^x)^6\left(\frac{1}{4^x}\right)^2}{{}^8C_1(2^x)^7\left(\frac{1}{4^x}\right)^1} = \frac{{}^8C_2}{{}^8C_1.2^x.4^x} = \frac{8x7}{2} \times \frac{1}{8} \times \frac{1}{8^x} = \frac{7}{2} \times \frac{1}{8^x}$$

When $x = -\frac{1}{3} \Rightarrow \quad \frac{T_3}{T_2} = \frac{7}{2} \times 8^{1/3} = 7$

When $x = \frac{1}{3}$

$\frac{T_3}{T_2} = \frac{7}{2} \times 8^{-1/3} = \frac{7}{4}$

18.5 (1)

Sol. $(x^8+1)^{60}\left\{\left(x^4+\frac{1}{x^4}\right)^3\right\}^{-10}$

$(x^8+1)^{60}\left(x^4+\frac{1}{x^4}\right)^{-30}$

$\frac{(x^8+1)^{60}(x^8+1)^{-30}}{x^{-120}}$

$x^{120}(1+x^8)^{30}$

Now coeff of $x^{160} = {}^{30}C_5 = \frac{30\times29\times28\times27\times26}{120} = 142506$

18.6 (2)

Sol. $\left[(ab^{1/2}+1)^4\right]^{20} = (ab^{1/2}+1)^{80}$

Hence number of dissimilar terms is 81

18.7 (3)

Sol. $T_{r+1} = {}^{100}C_r\,(\sqrt{2})^{100-r}\,(3)^{r/3}$

rational terms are when r = 0, 6, 12, 18, ..., 96

hence number of rational terms are 17

18.8 (3)

Sol. Here for greatest coefficient of x

= greatest term when x = 1

$\therefore r \le \frac{n+1}{1+\left|\frac{a}{b}\right|}$

$r \le \frac{50+1}{1+\left|\frac{3}{2x}\right|} = \frac{51}{5}\times 2$ as x = 1

$r \le \frac{102}{5}$

$r \le 20.4$

Hence T_{21} has greatest coefficient of x

$\therefore\ T_{21} = {}^{50}C_{20}\,.\,(3)^{50-20}\,.\,(2x)^{20}$

Now coeff. of x. = ${}^{50}C_{20} \times 3^{30} \times 2^{20}$

18.9 (3)

Sol. $r \le \frac{n+1}{1+\left|\frac{a}{b}\right|}$

$r \le \frac{18}{1+\left|\frac{3}{4x}\right|} = \frac{18}{1+\frac{1}{2}} = \frac{18\times2}{3} = r \le 12$

Since it is an integer

$\therefore T_{12}$ and T_{13} both are numerically greatest term and both are equal in magnitude

18.10 (4)

Sol. $(3^4)^{22} . 3^3 = (81)^{22} . 27$

$= 27(80 + 1)^{22} \quad = 27.80 \left[{}^{22}C_0 \cdot (80)^{21} + {}^{22}C_1 \cdot (80)^{20} \cdot (1) + ... {}^{22}C_{21}\right] + {}^{22}C_{22} \cdot 27$

Hence remainder is 27

18.11 (3)

Sol. 5^{5^5} is an odd natural number

Therefore $x = 5^{5^{5^5}} = 5^{2m+1} = 25^m.5$

where m is natural number

$\therefore \quad x = (24 + 1)^m . 5 = 5 +$ a multiple of 24

Hence, remainder is 5.

18.12 (1)

Sol. $2C_0 + 2^2 . \frac{C_1}{2} + 2^3 \frac{C_2}{3} + ... + 2^{n+1} \frac{C_n}{n+1} = \sum_{r=0}^{n} 2^{r+1} . \frac{{}^nC_r}{r+1} = \frac{1}{n+1} \sum_{r=0}^{n} 2^{r+1} . {}^{n+1}C_{r+1}$

$= \frac{1}{n+1} \cdot \left[(1+2)^{n+1} - 1\right] = \frac{3^{n+1} - 1}{n+1}$

18.13 (3)

Sol. $\sum_{r=0}^{10} (20 - 3r) \cdot {}^{10}C_r = 20 \sum_{r=0}^{10} {}^{10}C_r - 3 \sum_{r=0}^{10} r \cdot \frac{10}{r} . {}^9C_{r-1} = 20.2^{10} - 30 \sum_{r=1}^{10} {}^9C_{r-1} = 20.2^{10} - 30.2^9 = 2^9 . 10$

18.14 (2)

Sol. $\sum_{r=0}^{10} (63 - 10r)\, {}^{21}C_{2r} = 63 \cdot \sum_{r=0}^{10} {}^{21}C_{2r} - 5 \sum_{r=0}^{10} \frac{21}{2r} \cdot 2r\ {}^{20}C_{2r-1}$

$= 63 . 2^{21-1} - 5.21\ [2^{20-1}] = 63 . 2^{20} - 105 . 2^{19} = 2^{19} (126 - 105) = 2^{19} . 21$

18.15 (2)

Sol. $\frac{1}{n+1} [{}^{n+1}C_1.2 + {}^{n+1}C_2.2^2 + ... + {}^{n+1}C_{n+1}.2^{n+1}] = \frac{1}{n+1} \cdot \left[(1+2)^{n+1} - 1\right] = \frac{3^{n+1} - 1}{n+1}$

18.16 (2)

Sol. $(1 + x)^{11} = {}^{11}C_0 + {}^{11}C_1 . x + {}^{11}C_2 . x^2 + ... + {}^{11}C_{11} . x^{11}$

and $\quad (x + 1)^{11} = {}^{11}C_0 . x^{11} + {}^{11}C_1 . x^{10} + {}^{11}C_2 . x^9 + ... + {}^{11}C_{11} . 1$

compare x^8 in product

${}^{22}C_8 = {}^{11}C_0 . {}^{11}C_3 + {}^{11}C_1 . {}^{11}C_4 + ... + {}^{11}C_8 . {}^{11}C_{11}$

18.17 (4)

Sol. ${}^{50}C_{50} + {}^{51}C_{50} + ... + {}^{100}C_{50} = {}^{51}C_{51} + {}^{51}C_{50} + ... + {}^{100}C_{50} = {}^{101}C_{51}$

18.18 (1)

Sol. $\sum_{r=0}^{n-1} \left(\frac{{}^nC_r}{{}^{n+1}C_{r+1}} \right) = \sum_{r=0}^{n-1} \frac{r+1}{n+1} = \frac{\frac{n(n+1)}{2}}{n+1} = \frac{n}{2}$

18.19 (3)

$(3-2x)^{-3/4} = 3^{-3/4}\left(1-\frac{2x}{3}\right)^{-3/4}$

coeff. of $x^4 = \dfrac{\left(\frac{-3}{4}\right)\left(\frac{-3}{4}-1\right)\left(\frac{-3}{4}-2\right)\left(\frac{-3}{4}-3\right)\left(\frac{-2}{3}\right)^4}{4!} \times 3^{-3/4} = \frac{3}{4}\cdot\frac{7}{4}\cdot\frac{11}{4}\cdot\frac{15}{4}\cdot\frac{16}{3^4}\cdot\frac{1}{4!} = \frac{385}{128.3^3} \times 3^{-3/4}$

18.20 (1)

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}n^2 + ...$

Here comparing

$nx = \frac{7}{24}$ and $\frac{n(n-1)}{2}x^2 = 2.\frac{49}{576}$

by solving we get $x = -\frac{7}{8}$

and $n = -\frac{1}{3}$

$\therefore$ required sum $= \left(1-\frac{7}{8}\right)^{-1/3} = 2$

18.21 (2)

Sol. General term $= \frac{7!}{r_1!r_2!\,r_3!}(3xy)^{r_1}(-2xz)^{r_2}(zy)^{r_3}$

Here, $r_1 + r_2 + r_3 = 7$

$r_1 + r_2 = 6$

$r_1 + r_3 = 5$

$r_2 + r_3 = 3$

solving $r_1 = 4,\ r_2 = 2,\ r_3 = 1$

Hence coeff. $= \frac{7!}{4!\,2!\,1!}\cdot 3^4 . 2^2 . 1$

18.22 (3)

Sol. Number of terms $= {}^{70+3-1}C_{70}$

18.23 (2)

Sol. $\sum_{j=1}^{m}\sum_{i=1}^{n} i.j^2 = \sum_{j=1}^{m} j^2 \sum_{i=1}^{n} i = \sum_{j=1}^{m} j^2 \frac{(n)(n+1)}{2} = \frac{n(n+1)}{2}\sum_{j=1}^{m} j^2 = \frac{n(n+1)}{2}\cdot\frac{m(m+1)(2m+1)}{6}$

18.24 (1)

Sol. Put $x = 1$

$2^{20} = a_0 + a_1 + a_2 + ... + a_{40}$

where $a_0 = 1$ and $a_{40} = (-2)^{20}$

$\therefore$ $a_1 + a_2 + ... + a_{39} = 2^{20} - a_0 - a_{40} = 2^{20} - 1 - 2^{20} = -1$

18.25 (3)

Sol. $= 3.\left[{}^nC_1\cdot\frac{2}{3} + {}^nC_2\left(\frac{2}{3}\right)^2 + {}^nC_3\cdot\left(\frac{2}{3}\right)^2 + ... + {}^nC_n\left(\frac{2}{3}\right)^n\right] = 3.\left[\left(1+\frac{2}{3}\right)^n - 1\right] = 3\cdot\frac{5^n}{3^n} - 3 = \frac{5^n - 3^n}{3^{n-1}}$

18.26 (4)

Sol. $n(n^2-1) = (n-1)\,n(n+1)$ is divisible by 3

Let $n = 2m + 1$ then

$n(n^2-1) = (2m)(2m+1)(2m+2) = 4m(m+1)(2m+1)$ is divisible by 8

hence greatest number is 24

$= 2^9 . 10$

18.27 (2)

Sol. For S(1) $\Rightarrow$ $2 = -1 + (1 + 1)$

$2 = 1$ not true

Now let S(k) is true then

$2 + 4 + 6 + ... + 2k = -1 + k(k + 1)$

Now for S(k + 1)

$2 + 4 + 6 + ... 2k + 2(k + 1) = -1 + k(k + 1) + 2(k + 1) = -1 (k + 1) (k + 2)$

Hence S(k + 1) is also true

18.28 (2)

Sol. $a_1 = \sqrt{11} < 11$

Let $a_m < 11$

then $a_{m+1} = \sqrt{11 + a_m}$

$\Rightarrow a^2_{m+1} = 11 + a_m < 11 + 11 < 22 \quad \Rightarrow \quad a_{m+1} < \sqrt{22} < 11$; so by the principle of induction $a_n < 11 \;\forall n$

18.29 (1)

Sol. Let S(k) is true then

$S(k + 1) = 2.7^{k+1} + a.5^{K+1} - 5 \quad = 7(2.7^k + a.5^k - 5) + 30 - 2a.5^k \; = 7(2.7^k + a.5^k - 5) - 30\left(\frac{a.5^{k-1}}{3} - 1\right)$

for divisible by 24 $\quad a = 3$

18.30 (3)

Sol. P(3) is false, P(5) is false

Now Let P(m) is true

then $2^{m-2} > 3m$

Now $2^{m-2} \times 2 > 6m$

$2^{(m+1)-2} > 3m + 3m$

$2^{(m+1)-2} > 3m + 3$ [as $3m \ge 3$]

$2^{(m+1)-2} > 3(m + 1)$

18.31 (2)

Sol. If n is odd then it is divisible by $x - y$

and if n is even then it is divisible by $(x + y)(x - y)$

18.32 (4)

Sol. $10^{2n-a} + b = 11\lambda$, where λ = Integer

Now, $10^{2(n+1)-a} + b = 10^{2n-a} . 100 + b$

$= 100(10^{2n-a} + b) - 99b \quad = 100 . 11\lambda - 99b = 11 \times$ (Integer), for all a, b $\in$ N

18.33 (3)

Sol. $$\frac{1}{(n+4)(n+5)}\sum_{r=1}^{n} {}^{n+5}C_{r+2}$$

$$= \frac{1}{(n+4)(n+5)} \cdot \left[2^{n+5} - 2.\left(1 + (n+5) + \frac{(n+5)(n+4)}{2}\right)\right] = \frac{2^{n+5} - 2}{(n+4)(n+5)} - \frac{2}{n+4} - 1$$

18.34 (3)

Sol. ${}^{10}C_0$. [coeff. of x^{10} in $(1 + x)^{20}$] – ${}^{10}C_1$ [coeff. of x^{10} in $(1 + x)^{18}$] + ...

= coeff. of x^{10} in $[{}^{10}C_0 . (1 + x)^{20} - {}^{10}C_1 . (1 + x)^{18} + ...]$

= coeff. of x^{10} in $[(1 + x)^2 - 1]^{10}$ = coeff. of x^{10} in $[x^{10} (2 + x)^{10}] = 2^{10}$

18.35 (1)

Sol. $$\sum_{m=0}^{n} {}^{2n}C_{2m} \cdot \sum_{p=0}^{m} {}^{2m}C_{2p}$$

$$\sum_{m=0}^{n} {}^{2n}C_{2m} \cdot 2^{2m-1} = \frac{1}{2}\sum_{m=0}^{n} {}^{2n}C_{2m} \cdot 2^{2m} = \frac{1}{2}\left[\frac{1}{2}\left\{(1+2)^{2n} + (1-2)^{2n}\right\}\right] = \frac{1}{4}(3^{2n} + 1)$$

18.36 (1)

Sol. $\left(\frac{x-3}{-4}\right)^{5/7} = \left(\frac{-4}{x-3}\right)^{-5/7} = \left[\frac{x-3-(x+1)}{x-3}\right]^{-5/7} = \left[1-\frac{x+1}{x-3}\right]^{-5/7} = 1 + \frac{5}{7}\left(\frac{x+1}{x-3}\right) + ...\infty$

for this $\left|\frac{x+1}{x-3}\right| < 1$ Hence $x \in (-\infty, 1)$

18.37 (4)

Sol. Let $11^{n+2} + 12^{2n+b}$ is divisible by 133

Now $S(n+1) = 11^{n+2} . 11 + 12^{2n+b} . 12^2 = 11(11^{n+2} + 12^{2n+b}) + 133.12^{2n+b}$

Hence for n = 3, 2, 5, S(n + 1) is true

18.38 (4)

Sol. Since it is true for n = 1

Now let it is true for n = k

then $S(k+1) = \frac{d^{k+1}}{dx^{k+1}}\left(\frac{\log x}{x}\right) = \frac{d}{dx}\left(\frac{d^k}{dx^k}\left(\frac{\log x}{x}\right)\right) = \frac{d}{dx}\left[\frac{(-1)^k}{x^{k+1}}k!\left(\log x - 1 .... - \frac{1}{k}\right)\right]$

$= \frac{(-1)^k k!(-1)(k+1)}{x^{k+2}}\left(\log x - 1 - \frac{1}{2} .... - \frac{1}{k}\right) + \frac{(-1)^k k!}{x^{k+1}}.\frac{1}{x}$

$= \frac{(-1)^{k+1}(k+1)!}{x^{k+2}}\left(\log x - 1 - \frac{1}{2} .... - \frac{1}{k} - \frac{1}{k+1}\right)$

Hence none of the option is incorrect

18.39 (3)

Sol. for n = 1 S(1) is true

now let for n = k, S(n) is true

then $\frac{k^7}{7} + \frac{k^5}{5} + \frac{2}{3}k^3 - \frac{k}{105} = \lambda$ is an integer

then

$S(k+1) = \frac{(k+1)^7}{7} + \frac{(k+1)^5}{5} + \frac{2}{3}(k+1)^3 - \frac{k+1}{105}$

$= \frac{1}{7}(k^7 + 7k^6 + 21k^5 + ... + 7k + 1) + \frac{1}{5}(k^5 + 5k^4 + ... + 1) + \frac{2}{3}(k^3 + 3k^2 + 3k + 1)$

$- \frac{k}{105} - \frac{1}{105} = \left[\frac{k^7}{7} + \frac{k^5}{5} + \frac{2k^3}{3} - \frac{k}{105}\right] + k^6 + 3k^5 + 6k^4 + 7k^3 + 7k^2 + 4k + 1 = \lambda + \text{Integer} = \text{Integer}$

18.40 (3)

$\left(x + \frac{1}{x} + 2\right)^{21} = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^{42} \Rightarrow T_{r+1} = {}^{42}C_r (\sqrt{x})^{42-r}\left(\frac{1}{\sqrt{x}}\right)^r = {}^{42}C_r . x^{21-r}$

For independent of x, 21 – r = 0

r = 21

$T_{21+1} = {}^{42}C_{21}$ it is also middle term

but it is not always true

18.41 (1)

$2^{4n} - 2^n(7n+1)$

$= 2^n(2^{3n} - (7n+1))$

$= 2^n(1 + {}^nC_1 . 7 + ... + {}^nC_n . 7^n - 7n - 1) = 2^n . 7^2 . ({}^nC_2 + {}^nC_3 . 7 + ...)$

$= (2^{n-2}) 196({}^nC_2 + ...)$

19. Permution & Combination

19.1 (1)

Since a voter may chose one, two, three or four candidates out of 10, therefore the number of ways in which he can vote is

$^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 385$

19.2 (3)

$\lfloor 3\ (3 + 4 + 5 + 6) = 6\ (18) \equiv 108$

19.3 (4)

No. of ways = $\dfrac{\lfloor 9}{\lfloor 3 \lfloor 6} \times 2 = \dfrac{9 \times 8 \times 7}{6} \times 2 = 3 \times 56 = 168$

19.4 (2)

Number of triplets of positive integers which are solutions of $x + y + z = 100$

$x + y + z = 100$ $\qquad x \geq 1,\ y \geq 1,\ z \geq 1$

$(t_1+1) + (t_2+1)\ (t_3+1) = 100$ $\qquad x - 1 \geq 0,\ y - 1 \geq 0,\ z - 1 \geq 0$

$t_1 + t_2 + t_3 = 97$ $\qquad t_1 \geq 0,\ t_2 \geq 0,\ t_3 \geq 0$

Number of solutions = $^{97+3-1}C_{3-1} = {}^{99}C_2 = 4851$

19.5 (1)

No. of 5 letter words when one or more of its letters is repeated = 10^5

No. of 5 letter words when none of their letters repeated = $^{10}P_5 = 30240$

No. of 5 letter words when atleast one of their letters repeated = 100000 – 30240 = 69760

19.6 (4)

$^4C_2 \times {}^5C_3 \times 5! = 7200$

19.7 (2)

No. of triangles = $^{10}C_2 \times {}^{10}C_1 + {}^{10}C_1 \times {}^{10}C_2 = 900$

19.8 (1)

Words begining with A, D, M, N and O are 5! each. Words begining with RAD are 3! and also for RAM are 3!. Then comes RAND. First we shall have RANDMO and then RANDOM.

$\therefore$ rank is 5(5!) + 2(3!) + 2 = 614

19.9 (2)

$\dfrac{n-r+1}{r} = \dfrac{9}{2}$ and $\dfrac{n-r}{r+1} = \dfrac{8}{3} \Rightarrow \dfrac{8}{3}(r+1) + 1 = \dfrac{9r}{2} \Rightarrow r = 2$

r of ways in which

60

also for RAM

19.10 (1)

1, 2, 2

1,2,2 : $\frac{3P_3}{2!}$ way, 1 way (3) $\qquad \frac{{}^3P_3 \times 1}{2!} = 3$

2,2,3 : $\frac{{}^3P_3}{2!}$ ways, 1 way (1) $\qquad \frac{{}^3P_3}{2!} = 3$ ways

1,2,3 : ${}^3P_3$, 1 way (2)

3! ways = 6

∴ Sum of the digits at units, thens, hundredth and thosands places will be each

$3 \times 3 + 3 \times 1 + 6 \times 2 = 24$

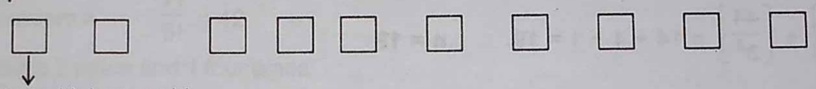
∴ Sum of numbers formed $= 24 \times 1 + 24 \times 10 + 24 \times 100 + 24 \times 1000 = 26664$

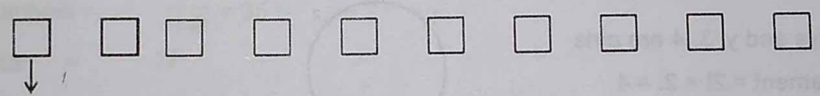
19.11 (2)

× U × CC × E or U × C C × E ×

$3! \times {}^4C_3 = 6 \times 4 = 24$

19.12 (1)





No.of ways $= 2 \times 10! \times 10!$

19.13 (3)

$9600 = 2^7 \times 3 \times 5^2$

∴ No. of divisors $= (7 + 1) \times (1 + 1) \times (2 + 1) = 8 \times 2 \times 3 = 48$

19.14 (1)

Coeff. of t^{30} in : $[1 + t + t^2 \ldots.. + t^{30}]^4$

: $(1 - t^{31})^4 (1 - t)^{-4}$

: $(1 - t)^{-4}$

: ${}^{4+30-1}C_{30}$

: ${}^{33}C_3 = \frac{33 \times 32 \times 31}{6} = 5456$

19.15 (1)

Treat india and pakistan prime ministers as one (I, P) + 7 others we have to arrange 8 person round a table no. of ways = 7! but corresponding to each arrangement india and pakistan primeministers interchange their places in 2! ways

Total ways = 7!.2!

19.16 (1)

5 Circles intersect in maximum $2 \times {}^5C_2$ points and 4 straight lines intersect in ${}^4C_2$ points and line and circle cut maximum in $2 \times {}^5C_1 \times {}^4C_1$ point.

Total maximum points is = 20 + 6 + 40 = 66 ways.

19.17 (1)

$2^5 \times 2^4 \times 2^3 - 1 = 2^{12} - 1 = 4095$

19.18 (1)

$(5 + 1)(4 + 1)2^3 - 1 = 239$

19.19 (2)

$$\text{Grouping} = \frac{5!}{2!\,2!\,1!\,2!} + \frac{5!}{3!\,1!\,1!\,2!} = 25$$

Distribution to 4 students = 25 × 4! = 600

19.20 (1)

$5^4 = 625$

19.21 (1)

$$4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$$

19.22 (4)

$({}^nC_1 \times {}^nC_2) + ({}^nC_2 \times {}^nC_1) = n^2(n-1)$

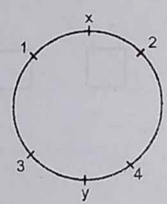
19.23 (2)

$$\left[\frac{44}{3}\right] + \left[\frac{44}{3^2}\right] + \left[\frac{44}{3^3}\right] = 14 + 4 + 1 = 19 \qquad \mathbf{n = 19}$$

19.24 (1)

x, 1, 2 are boys and y, 3, 4 are girls

No. of arrangement = 2! × 2! = 4



19.25 (1)

No. of possible tickets $= {}^{14-1}C_2 = 78$

No. of selection of 75 ticket from 78 tickets $= {}^{78}C_{75} = {}^{78}C_3$

19.26 (1)

Case - I when all 5 match win the india then total

no. of ways $= {}^5C_5$

Case - II when India wins 6th match then total

no. of ways $= {}^5C_4$

Case - III when India wins 7th match then total

no. of ways $= {}^6C_4$

Case - IV when India wins 8th match then total

no. of ways $= {}^7C_4$

Case - V when India wins 9th match then total

no. of ways $= {}^8C_4$

Hence required no. of ways

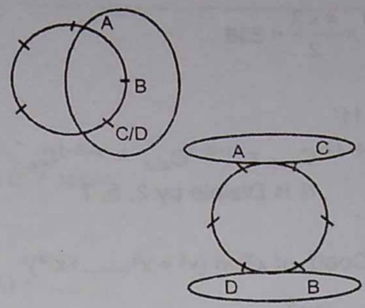
$= 1 + 5 + {}^6C_4 + {}^7C_4 + {}^8C_4 = 1 + 5 + 15 + 35 + 70 = 126$

19.27 **(3)**

Case-I If B is right on A

Subcase -I C is right on B

then no. of ways = (4 –1)! = 6



Subcase- II If D is right on B

then no. of ways = (4 –1)! = 6

Case-II If C is right on A

⇒ D must be right on B

= (4 – 1)! = 3! = 6

Hence total no. of ways is 6 + 6 + 6 = 18

19.28 **(3)**

SERIES

S - 2, E - 2 , R, I

case-I when all letter distinct is

$^4C_3 \times 3! = 4 \times 6 = 24$

case-II when 2 letters are same the

$^2C_1 \,.\, ^3C_1 \times \frac{3!}{2!} = 2 \,.\, 3 \,.\, 3 = 18$

total number is 24 + 18 = 42

19.29 **(3)**

Taking 3 once, 2 once and 1 five times

Numbers = $\frac{7!}{5!} = 42$

Taking 2 thrice and 1 four times

Numbers = $\frac{7!}{3!\,4!} = 35$

Total = 77

19.30 **(1)**

Coeff. of x^4 in : $(1 + x + x^2 + x^3 + x^4 + x^5)(1 + x + x^2 + x^3 + x^4)(1 + x + x^2)$

: $(1 - x^6)(1 - x^5)(1 - x^3)(1 - x)^{-3}$

: $[1 - x^3][1 - x]^{-3}$

: $[^{3+4-1}C_4 - ^{3+1-1}C_1]$

: 15 – 3

: 12

19.31 **(2)**

Total = $^9C_2 \times {^9C_2} = 36 \times 36 = 1296$

Square = $8^2 + 7^2 \ldots\ldots + 1^2 = \frac{8.9.17}{6} = 204$

Rectangle excluding squares = 1296 – 204 = 1092

19.32 **(2)**

Coeff of t^6 : $[^4C_1t + {^4C_2}t^2 \ldots\ldots + {^4C_4}t^4]\,[^2C_1t + {^2C_2}t^2]^2$

$= [(1 + t)^4 - 1][(1 + t)^2 - 1]^2$

$= (1 + t)^8 - 2(1 + t)^6$

$= {^8C_2} - 2.{^6C_6} = 26$

No. of ways with arrangement = 26.6!

19.33 **(2)**

$^8C_3 \cdot {^4C_2} = \frac{8\times7\times6}{6}\times\frac{4\times3}{2} = 336$

19.34 **(4)**

$x_1x_2x_3x_4 = 2^4 \times 5^1 \times 11^1$

No. of solutions = $^{4+4-1}C_{4-1} \times {^{1+4-1}C_{4-1}} \times {^{1+4-1}C_{4-1}} = {^7C_3} \times {^4C_3} \times {^4C_3}$

$\Rightarrow N = 35 \times 4 \times 4$ N is Disible by 2, 5, 7

19.35 **(1)**

required number = Coeff. of x^{30} in $(x^2 + x^3 .... + x^{16})^8$

= Coeff. of x^{14} in $\left(\frac{1-x^{15}}{1-x}\right)^8$ = Coeff. of x^{14} is $(1-x)^{-8} = {^{21}C_7}$

19.36 **(4)**

$x + y + z \le n$

$x + y + z + t = n$

when $t \ge 0$

no. of solutions

$^{n+4-1}C_{4-1} = {^{n+3}C_3}$

reason is true

19.37 **(2)**

Obvious

19.38 **(2)**

Point of intersection of 8 unequal circles =	$2 . {^8C_2}$ =	$2 \times \frac{8\times7}{2}$	=	56
Point of intersection of 4 unequal circles	=	$2 . {^4C_2}$	=	12
Point of intersection of 4 non coincident line	=	$1 . {^4C_2}$	=	6
Point of intersection of 4 circles and 4 lines	=	$2 . {^4C_1} . {^4C_1}$	=	32
				50

19.39 **(3)**

$a + b + c \le 8$

$a = x + 1$

$b = y + 1$

$c = z + 1$

$x + y + z \le 5$

$x + y + z + w = 5$

No. of non negative integral solutions = $^8C_3 = \frac{8\times7\times6}{6} = 56$

$x_1 + x_2 + ... + x_r = n$

No. of non negative integral solution = $^{n+r-1}C_{r-1}$

20. Probability

20.1 (3)

Let, event A : sum of 7 occurs

even B : sum of 5 occurs

event C : nseither sum of 5 nor sum of 7 occur

$\Rightarrow P(A) = \frac{1}{6}, P(B) = \frac{1}{9}, P(C) = \frac{13}{18}$

$\Rightarrow P = P(A) + P(C) \cdot P(A) + P(C)^2 P(A) + \ldots\ldots$

$= \frac{P(A)}{1-P(C)} = \frac{3}{5}. \Rightarrow 5P = 3$

20.2 (2)

Let E_1 = it is from bag A

E_2 = it is from bag B

E = ball is white

$P(E_1) = P(E_2) = \frac{1}{2}$ ....(1)

$P\left(\frac{E}{E_1}\right) = \left(\frac{5}{8}\right)$ ....(2)

$P\left(\frac{E}{E_2}\right) = \frac{3}{4}$ ....(3)

$$P\left(\frac{E_2}{E}\right) = \frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_2)P\left(\frac{E}{E_2}\right)+P(E_1)P\left(\frac{E}{E_1}\right)} = \frac{3/4}{\frac{3}{4}+\frac{5}{8}} = \frac{6}{11}$$

20.3 (1)

Total no. of ways in which 6 persons can have their birth days = 12×12×12×12×12×12 = 12^6 out of 12 months , 2 month can be chosen in ${}^{12}C_2$ ways. Now birth days of six persons can fall in these two months in 2^6 ways. Out of these 2^6 ways, there are two ways when all six birth days fall in one month. So there are $(2^6 - 2)$ ways in which six birth days fall in chosen 2 months.

$\therefore$ Required probability = $\frac{{}^{12}C_2(2^6-2)}{12^6} = \frac{341}{12^5}$

20.4 (3)

Order of matrix A can be 6 ways,

1 × 32 , 2 × 16 , 4 × 8 , 8 × 4 , 16 × 2 , 32 × 1

Order of matrix B can be 8 ways

1 × 56 , 2 × 28 , 4 × 14 , 7 × 8 , 8 × 7 , 14 × 4 , 28 × 2 , 56 × 1

Total number of ways of forming A and B both = 48 ways

Now no. of ways in which A × B is possible is only 4.

$\therefore$ probability = $\frac{4}{48} = \frac{1}{12}$

20.5 (3)
Number of ways of choosing A and B = $2^n.2^n = 2^{2n}$
The number of subsets which contain exactly r elements is ${}^nC_r$.
∴ Number of ways to choose A and B such that they have same number of elements is -

$= ({}^nC_0)^2 + ({}^nC_1)^2 + ({}^nC_2)^2 + ... + ({}^nC_n)^2 = {}^{2n}C_n$

∴ required probability $= \frac{{}^{2n}C_n}{2^{2n}} = \frac{1.3.5......(2n-1)}{2^n\, n!}$

20.6 (3)
□ □ □ □
Odd digit = 1, 3, 5
Number of ways of end digits = ${}^3C_2 \times 2! = 6$
No. of ways of remaining two places = ${}^5C_2 \times 2! = 20$
Total ways = 6 × 20 = 120

Probability $= \frac{120}{{}^7C_4 \times 4!} = \frac{1}{7}$

20.7 (2)
Last two digits are = 16, 24, 32, 36, 52, 56, 64, 72, 76, 12

Probability $= \frac{{}^5C_3 \times 3! \times 10}{{}^7C_5 \times 5!} = \lambda = \frac{20}{3}$

20.8 (2)
□□□□□□□□□
Vowels = 4
Consonants = 5

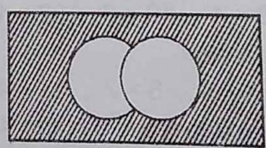
$\frac{4! \times 5!}{9!} = \frac{1}{126}$ (4 places for vowels and 5 for consonants)

20.9 (1)
A = T1, T3, T5
B = H2, H4, H6

Probability (A or B) $= \frac{6}{12} = \frac{1}{2}$

Sample space of (A and B) = 0

20.10 (3)
$P(A + B) = P(A) + P(B) - P(A \cap B)$
$= 0.42 + 0.48 - 0.16 = 0.74$
$P(\overline{A+B}) = 1 - 0.74 = 0.26$



20.11 (3)

Probability of randomly selected month $= \frac{1}{12}$

Probability of 13th day is a saturday = 1/7

Required probability $= \frac{1}{84}$

nts is -

20.12 **(2)**
Possible out comes = 36
Favourable cases = (2, 5), (4, 3) (6, 1)

Probability = $\frac{3}{36}$ [dependent event] = $\frac{1}{12}$

20.13 **(4)**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{23}{60}$$

$$P\left(\frac{\bar{B}}{\bar{A}}\right) = \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})} = \frac{1 - P(A \cup B)}{P(\bar{A})} = \frac{1 - \frac{23}{60}}{1 - \frac{1}{3}} = \frac{37}{40}$$

20.14 **(3)**

Probability of occurance $= \frac{4}{9}$

Probability of non occurance $= 1 - \frac{4}{9} = \frac{5}{9}$

20.15 **(3)**

$$\text{Mean} = \sum x_i p(x_i) = 1.\frac{1}{10} + 2.\frac{2}{10} + 3.\frac{3}{10} + 4.\frac{4}{10} = \frac{4(4+1)(8+1)}{6.10} = 3$$

20.16 **(2)**
Total = 9
Favourable cases = 5

Probability = $\frac{5}{9}$

20.17 **(2)**

$P[E_1] = \frac{3}{6} = \frac{1}{2}$ $P(E_2) = \frac{3}{6} = \frac{1}{2}$

P_1 (ball drawn from first box is white) = $\frac{4}{9}$

P_2 (ball drawn from 2nd box is white) = $\frac{5}{9}$

∴ By Bayer's theorm probability of ball draw from first box = $\dfrac{\frac{1}{2}.\frac{4}{9}}{\frac{1}{2}.\frac{4}{9} + \frac{1}{2}.\frac{5}{9}} = \frac{4}{9}$

20.18 **(2)**

$$P(\text{fresh egg}) = \frac{80}{100} = \frac{8}{10} = p$$

$$P(\text{rotten egg}) = \frac{20}{100} = \frac{2}{10} = q;\ n = 5,\ r = 5$$

so probability that none egg is

rotten = ${}^5C_5 \left(\frac{8}{10}\right)^5 \left(\frac{2}{10}\right)^0$

20.19 (2)

Probability of red card = $\frac{^{26}C_1}{^{52}C_1} = \frac{1}{2}$

Probability of C's winning

$= P(\overline{A})P(\overline{B}).P(C) + P(\overline{A})P(\overline{B})P(\overline{C})P(\overline{A})P(\overline{B})P(C) + \ldots\ldots$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \ldots\ldots$

$= \frac{1/8}{1-1/8} = \frac{1}{7}$

20.20 (3)

A $\rightarrow$ Ball drawn is black; $E_1 \rightarrow$ Bag I is chosen
$E_2 \rightarrow$ Bag II is chosen and $E_3 \rightarrow$ Bag 3rd is chosen

Then $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P(A/E_1) = \frac{3}{5}$ $\quad P(A/E_2) = \frac{2}{6} = \frac{1}{3}$

$P(A/E_3) = \frac{2}{5}$

Required probabilty = $\frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{5} = \frac{1}{3}\left[\frac{3}{5}+\frac{1}{3}+\frac{2}{5}\right] = \frac{1}{3}\times\frac{4}{3} = \frac{4}{9}$

20.21 (4)

$$^5C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2 + {}^5C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^1 + {}^5C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^0 = \frac{192}{243}$$

20.22 (1)

$P(A) = \frac{3}{4}$ (he knows the answer)

$P(B) = \frac{1}{4}$ (he does not know the answer)

$P(C/A) = 1$ (he gets correct answer and knows the correct answer)

$P(C/B) = \frac{1}{4}$ (he get the correct answer but does not know)

Required probability = $\frac{P(B).P(C/B)}{P(A)P(C/A)+P(B)P(C/B)} = \frac{\frac{1}{4}.\frac{1}{4}}{\frac{3}{4}.1+\frac{1}{4}.\frac{1}{4}} = \frac{1}{13}$

20.23 (2)

Let probability of success is 'P'

$P(x = 3) = {}^8C_3 P^3 (1-P)^5$

$P'(X) = {}^8C_3 [3p^2(1-p)^5 - 5(1-P)^4 P^3]$

for maximum

$\Rightarrow 3(1-P) - 5P = 0$

$3 - 8P = 0 \Rightarrow P = 3/8$

Probability of failure = $\frac{5}{8}$

20.24 (1)

A_1 : He know the answer

A_2 : He does not know the answer

E : He gets the correct answer

$P(A_1) = \frac{9}{10}$ $\quad P(A_1) = 1 - \frac{9}{10} = \frac{1}{10}$

$P(E/A_1) = 1$ $\quad P(E/A_2) = \frac{1}{4}$

Required probability = $P(A_1)\,P(E/A_1) + P(A_2).P(E/A_2)$

$= \frac{9}{10} \times 1 + \frac{1}{10} \times \frac{1}{4} = \frac{36+1}{40} = \frac{37}{40}$

20.25 (3)

	P_i	M_i
(i) 5 one ruppes coin	$\frac{1}{32}$	5
(ii) 4 one rupees + one 2 rupees	$\frac{5}{32}$	6
(iii) 3 one rupees + two 2 rupees	$\frac{10}{32}$	7
(iv) 2 one rupees + 3 two rupees	$\frac{10}{32}$	8
(v) 1 one rupees + 4 two rupees	$\frac{5}{32}$	9
(vi) 5 two rupees coin	$\frac{1}{32}$	10

Expectation : $\sum P_i M_i$

$= \frac{5}{32} + \frac{30}{32} + \frac{70}{32} + \frac{80}{32} + \frac{45}{32} + \frac{10}{32} = 7.5$ rupees

20.26 (4)

$P(E \cap F) = 1/12$ and $P(E' \cap F') = 1/2$

As E and f are independent, we get

$P(E) . P(F) = 1/12$ and $P(E')\,P(F') = 1/2$

$\Rightarrow (1-P(E))\,(1-P(F)) = 1/2$

$\Rightarrow P(E) + P(F) = 7/12$

$\therefore$ Equation whose roots are p(E) and P(F) are

$x^2 - (P(E) + P(F))\,x + P(E) . P(F) = 0$

or $x^2 - \frac{7}{12}x + \frac{1}{12} = 0$

$12x^2 - 7x + 1 = 0$

$\Rightarrow (3x-1)\,(4x-1) = 0$

$\Rightarrow x = 1/3\ ,\ 1/4$

20.27 (1)

Let E denote the event that a six occurs & A the event that the man reports that it is a six .

$P(E) = 1/6$, $P(E') = 5/6$, $P(A/E) = 3/4$ & $P(A/E') = 1/4$

$$P(E/A) = \frac{P(E).P(A/E)}{P(E).P(A/E)+P(E')P(A/E')} = \frac{(1/6).(3/4)}{\left(\frac{1}{6}\right)\left(\frac{3}{4}\right)+\left(\frac{5}{6}\right)\left(\frac{1}{4}\right)} = \frac{3}{8}$$

20.28 (4)
Total no. of arrangements = 8 !
favourable cases = ${}^8C_3 \times 5!$

$$\text{probability} = \frac{{}^8C_3 \times 5!}{8!} = \frac{1}{3!} = \frac{1}{6}$$

20.29 (3)

$$\text{Discriminitant} = m^2 - 4\left(\frac{1}{2}+\frac{m}{2}\right) = m^2 - 2m - 2 = (m-1)^2 - 3$$

$D \ge 0 \Rightarrow (m-1)^2 \ge 3$
This is possible for m = 3 , 4 & 5 . Also , the total no. ways of choosing m is 5

$$\text{probaibility} = \frac{3}{5}$$

20.30 (3)

$$\text{Total ways} = {}^{2n-1}C_3 = \frac{(2n-1)(2n-2)(2n-3)}{1.2.3} = \frac{(2n-1)(n-2)(2n-3)}{3}$$

Favourable cases = (odd + odd) or (even + even)

$$= {}^nC_2 + {}^{n-1}C_2 = \frac{n(n-1)}{2} + \frac{(n-1)(n-2)}{2} = \frac{n-1}{2}(2n-2) = (n-1)^2$$

$$\therefore \text{ Required probability} = \frac{(n-1)^2 \times 3}{(2n-1)(n-1)(2n-3)} = \frac{3(n-1)}{(2n-1)(2n-3)}$$

20.31 (1)
$P(\overline{A} \cap B) = P(B) - P(A \cap B)$ is true
but by given information
$P(A \cap \overline{B}) = .3 - P(A \cap B)$
$\therefore$ $P(A \cap \overline{B})$ can not be found

20.32 (2)
As A , B, C are mutually independent
$P(B \cap C) = P(B)\,P(C)$, $P(C \cap A) = P(c)\,.\,P(A)$
$P(A \cap B) = P(A)\,.\,P(B)$, $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$
$P[A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$
$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) = P(A)[P(B) + P(C) - P(B)P(C)] = P(A)\,P(B \cup C)$
$P(A \cap B \cap C) = P(A)\,.\,P(B)\,P(C) = P(A)\,.\,P(B \cap C)$.

20.33 (3)
$P(A \cup B) = P(A \cap B)$
$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A \cap B)$
$\Rightarrow [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] = 0$ .... (i)
$P(A)$, $P(B)$ ³ $P(A \cap B)$
$\therefore$ (i) is possible if only
$P(A) - P(A \cap B) = 0$ $P(B) - P(A \cap B) = 0$
$\Rightarrow P(A) = P(B) = P(A \cap B)$
$\therefore$ $P(A) + p(B) = 1$ need not hold
next , $P(A \cap b') = P(A) - P(A \cap B) = 0$
similarly $P(A' \cap B) = P(B) - P(A \cap B) = 0$

20.34 (2)
Total number of favourable cases for drawing an ace or king = 8
Total number of cases for drawing a card = 52

$$\text{Probability} = \frac{8}{52} = \frac{2}{13}$$

Hence statement I^{st} is true
Statement 2^{nd} is true by formula

21. Matrices & Determinant

21.1 **(4)**

$$\Delta'(x) = \begin{vmatrix} 2x+4 & 2x+4 & 1 \\ 4x+5 & 4x+5 & 2 \\ 16x-6 & 16x-6 & 8 \end{vmatrix} + \begin{vmatrix} x^2+4x-3 & 2 & 1 \\ 2x^2+5x-9 & 4 & 2 \\ 8x^2-6x+1 & 16 & 8 \end{vmatrix}$$

$= 0 + 0 = 0$

$\Rightarrow \Delta(x) = \text{constant} = d$

so $a = 0$, $b = 0$, $c = 0$

Now $\Delta(0) = d = \begin{vmatrix} -3 & 4 & 1 \\ -9 & 5 & 2 \\ 1 & -6 & 8 \end{vmatrix} = -3(40+12) - 4(-72-2) + 1\,(54-5)$

$= -156 + 296 + 49 = 140 + 49 = 189$

21.2 **(2)**

$|A^{2005} - 6A^{2004}| = |A|^{2004}\ |A - 6I_2|$

$= (2)^{2004} \begin{vmatrix} 0 & 11 \\ 2 & -2 \end{vmatrix} = (-22)\,(2^{2004}) = (-11)\,(2^{2005})$

21.3 **(1)**

A28 , 3B9 , 62C are divisible by K, so

$A28 = (100A + 28) = n_1K \quad , n_1 \in N$

$3B9 = 300 + 10B + 9 = n_2K \ , \ n_2 \in N$

$62C = 620 + C = n_3K \ , n_3 \in N$

Now applying $R_2 \to R_2 + 100\,R_1 + 10R_3$

$$\Delta = \begin{vmatrix} A & 3 & 6 \\ 100A+28 & 309+10B & 620+C \\ 2 & B & 2 \end{vmatrix}$$

$$= \begin{vmatrix} A & 3 & 6 \\ n_1K & n_2K & n_3K \\ 2 & B & 2 \end{vmatrix} = K \begin{vmatrix} A & 3 & 6 \\ n_1 & n_2 & n_3 \\ 2 & B & 2 \end{vmatrix} = K(\text{an integer})$$

21.4 **(2)**

$\because 0 \le [x] < 2 \Rightarrow [x] = 0, 1$

$-1 \le [y] < 1 \Rightarrow [y] = -1, 0$

and $1 \le [z] < 3 \Rightarrow [z] = 1, 2$

Now $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$,

applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\begin{vmatrix} [x]+1 & [y] & [z] \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$

$= ([x]+1) - [y](-1) + [z](1)$

$= [x] + [y] + [z] + 1$

for maximum value of determinant $[x] = 1, [y] = 0, [z] = 2$

maximum value $= 1 + 0 + 2 + 1 = 4$

21.5 **(3)**

$$\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix} \times \begin{vmatrix} c & a & b \\ b & c & a \\ -a & -b & -c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = (3abc - a^3 - b^3 - c^3)^2 = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc \Rightarrow a + b + c = 0$$

$\therefore \quad 3(a + b + c) + 2 = 2$

21.6 **(1)**

$$\Delta = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$= (\lambda + 2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$= (\lambda + 2)(\lambda - 1)^2 = 0$

so $\lambda = -2$ or 1

so quadratic equation is $x^2 - (-2 + 1)x + (-2 \times 1) = 0$

$x^2 + x - 2 = 0$

21.7 **(2)** $g'(x) = f(x)$ (by lebnitz rule)

$= -\dfrac{1}{2} \sin 2x$ (by expansion of determinant)

so range of $g'(x) = \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

21.8 (1)

$$\sum_{r=1}^{n} \Delta_r = \begin{vmatrix} \sum_{r=1}^{n} r.r! & \sum_{r=1}^{n} {}^nC_r & 2\sum_{r=1}^{n} r \\ (n+1)! & 2^n & n^2+n+2 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} (n+1)!-1 & 2^n-1 & n(n+1) \\ (n+1)! & 2^n & n^2+n+2 \\ 1 & 1 & 2 \end{vmatrix}$$

$R_1 \to R_1 - R_2 + R_3$

$$= \begin{vmatrix} 0 & 0 & 0 \\ (n+1)! & 2^n & n^2+n+2 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

21.9 (4)

$$\sum_{r=1}^{n} \Delta_r = \begin{vmatrix} \frac{n(n-1)}{2} & n & 6 \\ \frac{n(n-1)(2n-1)}{6} & 2n^2 & 4n-2 \\ \frac{n^2(n-1)^2}{4} & 3n^3 & 3n^2-3n \end{vmatrix} = \frac{n^2(n-1)}{12} \begin{vmatrix} 6 & 1 & 6 \\ 4n-2 & 2n & 4n-2 \\ 3n^2-3n & 3n^2 & 3n^2-3n \end{vmatrix}$$

Here $C_1 = C_3$

so $\sum_{r=1}^{n} \Delta_r = 0$

21.10 (2)

$$\Delta = 2\begin{vmatrix} 1 & 1 & 1 \\ n & n+3 & n+6 \\ \frac{n(n-1)}{2} & \frac{(n+3)(n+2)}{2} & \frac{(n+6)(n+5)}{2} \end{vmatrix}$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$x = 2\begin{vmatrix} 1 & 0 & 0 \\ n & 3 & 6 \\ \frac{n(n-1)}{2} & 3(n+1) & \frac{12n+30}{2} \end{vmatrix}$$

$x = 2\,(27)$

$x = 2.3^3$

So number of relative prime factor $= 2^{2-1} = 2$

21.11 (2)

If A and B are two 3 × 3 order matrices

$(AB)' = B'A'$ [Reversal Law]

So $(AB)' = A'\,B'$ false

21.12 **(3)**

$A = -A'$

$\Leftrightarrow$ A is a skew symmetric matrix

$\Leftrightarrow$ diagonal elements are zero.

$x = 0,\ y = 0$

$x + y = 0$

21.13 **(2)**

As A is skew symmetric matrix

$A' = -A$

$\Rightarrow \quad a_{ii} = 0 \ \forall \ i \quad \Rightarrow \quad \text{trace}(A) = 0$

21.14 **(2)**

For the system of equation to have no solution we must have $\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$

$\Rightarrow \quad (k+1)(k+3) = 8k$ and $8(3k-1) \neq 4k(k+3)$

but $(k+1)(k+3) = 8k \quad \Rightarrow \quad k^2 + 4k + 3 = 8k$

or $\quad k^2 - 4k + 3 = 0$ or $(k-1)(k-3) = 0$

$k = 1,\ k = 3$

for $\quad k = 1,\ 8(3k-1) = 16$ and $4k(k+3) = 16$

for $\quad k = 3,\ 8(3k-1) = 64$ and $4k(k+3) = 72$

thus, for $k = 3,\ 8(3k-1) \neq 4k(k+3)$

21.15 **(4)**

The given system of equation will have a non-trivial solution if

$$\Delta = \begin{vmatrix} p\alpha + q & p & q \\ q\alpha + r & q & r \\ 0 & p\alpha + q & q\alpha + r \end{vmatrix} = 0$$

$R_3 \to R_3 - \alpha R_1 - R_2$

$$\begin{vmatrix} p\alpha + q & p & q \\ q\alpha + r & q & r \\ -(p\alpha^2 + 2q\alpha + r) & 0 & 0 \end{vmatrix} = 0$$

$-(p\alpha^2 + 2q\alpha + r)(pr - q^2) = 0$

So p, q, r in G.P or $p\alpha^2 + 2q\alpha + r = 0$

21.16 **(2)**

$A^4 = (5A - 7I)^2 = 25A^2 - 70A + 49I$

$= 25(5A - 7I) - 70A + 49I$

$= 55A - 126I$

Square both side

$A^8 = 3025A^2 - 13860A + 15876I$

$= 3025(5A - 7I) - 13860A + 15876I$

$= 1265A - 5299I$

So $\quad p = 1265$

21.17 **(2)**

$A' = -A$

$AA' = -A^2$

$AA' = I_n$

So A is orthogonal and $\det(AA') = \det I_n$

$(-1)^n \det(A^2) = 1$

$|A|^2 = 1$ if $n \to$ even

21.18 (2)

$|f(\alpha)| = \cos^2\alpha + \sin^2\alpha = 1$

$$\text{adj}(f(\alpha)) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(f(\alpha))^{-1} = \frac{\text{adj}(f(\alpha))}{|f(\alpha)|} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{so } f(-\alpha) = (f(\alpha))^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

21.19 (1)

$$f''(x) = \begin{vmatrix} 2 & 6x & 12x^2 \\ 2 & 3 & 6 \\ p & p^2 & p^3 \end{vmatrix}$$

$f''(x) = 0$

$\Rightarrow \quad 12(2p^2 - 3p)\, x^2 - 6x\,(2p^3 - 6p) + 6p(p^2 - 2p) = 0$

$\Rightarrow \quad 12p(2p - 3)\, x^2 - 12p\,(p^2 - 3)\, x + 6p\,(p^2 - 2p) = 0$

compare with $ax^2 + bx + c = 0$

at $p = 0$, its an identity $(a = b = c = 0)$

at $p = \sqrt{3}$, $\frac{b}{a} = 0$. So roots are of opposite in sign and equal in magnitude

at $p = 2$, $\frac{c}{a} = 0$, so product of roots is zero

at $p = -\sqrt{3}$, $\frac{c}{a} = \frac{6p(p^2 - 2p)}{12p(2p-3)} = -\text{ve}$

so product of roots are negative

21.20 (2)

$$\sum_{r=0}^{\infty} \Delta_r = \begin{vmatrix} \sum_{r=0}^{\infty} a^r & \sum_{r=0}^{\infty} b^r & \sum_{r=0}^{\infty} c^r \\ a & b & c \\ 1-a & 1-b & 1-c \end{vmatrix}$$

$$= \begin{vmatrix} \frac{1}{1-a} & \frac{1}{1-b} & \frac{1}{1-c} \\ a & b & c \\ 1-a & 1-b & 1-c \end{vmatrix}$$

its possible, where $|a|, |b|, |c| < 1$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} \frac{1}{1-a} & \frac{b-a}{(1-a)(1-b)} & \frac{c-a}{(1-c)(1-a)} \\ a & b-a & c-a \\ 1-a & a-b & a-c \end{vmatrix} = (b-a)(c-a) \begin{vmatrix} \frac{1}{1-a} & \frac{1}{(1-a)(1-b)} & \frac{1}{(1-c)(1-a)} \\ a & 1 & 1 \\ 1-a & -1 & -1 \end{vmatrix}$$

$C_3 \to C_3 - C_2$

$$= (b-a)(c-a) \begin{vmatrix} \frac{1}{1-a} & \frac{1}{(1-a)(1-b)} & \frac{c-b}{(1-c)(1-a)(1-b)} \\ a & 1 & 0 \\ 1-a & -1 & 0 \end{vmatrix} = \frac{(b-a)(c-a)(b-c)}{(1-a)(1-b)(1-c)}$$

Now $\sum_{r=0}^{\infty} \Delta_r = 0 \Rightarrow \frac{(b-a)(c-a)(b-c)}{(1-a)(1-b)(1-c)} = 0$

$a = b = c$ and $|a| < 1$

21.21 (1)

$AA' = I$ for orthogonal matrix

$$\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix} \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4b^2+c^2 & 2b^2-c^2 & -2b^2+c^2 \\ 2b^2-c^2 & a^2+b^2+c^2 & a^2-b^2-c^2 \\ 2b^2+c^2 & a^2-b^2-c^2 & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad b = \pm \frac{1}{\sqrt{6}}, \; a = \pm \frac{1}{\sqrt{2}}, \; c = \pm \frac{1}{\sqrt{3}}$$

21.22 (2)

$AB = A$ given

$\Rightarrow \quad ABA = A^2$

$A.(BA) = A^2$

$AB = A^2$

$A = A^2$

similarly $B = B^2$

21.23 (4)

$$\Delta = \begin{vmatrix} \frac{16}{8}\cdot{}^{15}C_7 & {}^{16}C_8 & {}^{15}C_8 \\ \frac{14}{7}\cdot{}^{13}C_6 & {}^{14}C_6 & {}^{13}C_5 \\ \frac{12}{6}\cdot{}^{11}C_5 & {}^{12}C_5 & {}^{11}C_4 \end{vmatrix} \quad \text{using } \left({}^{n}C_r = \frac{n}{r}\,{}^{n-1}C_{r-1}\right)$$

$$\Rightarrow \quad \Delta = 2\begin{vmatrix} {}^{15}C_7 & {}^{16}C_8 & {}^{15}C_8 \\ {}^{13}C_6 & {}^{14}C_6 & {}^{13}C_5 \\ {}^{11}C_5 & {}^{12}C_5 & {}^{11}C_4 \end{vmatrix}$$

$C_1 \to C_1 + C_3$ and using ${}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r$

$$\Rightarrow \quad \Delta = \begin{vmatrix} {}^{16}C_8 & {}^{16}C_8 & {}^{15}C_8 \\ {}^{14}C_6 & {}^{14}C_6 & {}^{13}C_5 \\ {}^{12}C_5 & {}^{12}C_5 & {}^{11}C_4 \end{vmatrix}$$

Here $C_1 = C_2$

So $\Delta = 0$

Now $\quad 2\,.\,{}^{13}C_6 - {}^{14}C_7 = \frac{14}{7}\,.\,{}^{13}C_6 - {}^{14}C_7 = {}^{14}C_7 - {}^{14}C_7 = 0$

21.24 (1)

Given system can be written as

$(a + b + c)(y + z) - a(x + y + z) = b - c$ ....(i)

$(a + b + c)(x + z) - b(x + y + z) = c - a$ ....(ii)

$(a + b + c)(x + y) - c(x + y + z) = a - b$ ....(ii)

(i) + (ii) + (iii)

$2(a + b + c)(x + y + z) - (x + y + z)(a + b + c) = 0$

$(a + b + c)(x + y + z) = 0$ or $(x + y + z) = 0$

so $x + y = -z$

by (iii)

$(a + b + c)(-z) - 0 = a - b$

$$z = \left(\frac{b-a}{a+b+c}\right)$$

similarly $x = \frac{c-b}{a+b+c}$ and $y = \frac{a-c}{a+b+c}$

so unique solution

21.25 (3)

Here $\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

$$\text{L.H.S.} = \begin{pmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{pmatrix}\begin{pmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{pmatrix}^{-1} = \cos^2\theta\begin{pmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{pmatrix}\begin{pmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{pmatrix}$$

$$= \cos^2\theta\begin{pmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{pmatrix} = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\text{R.H.S.} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

L.H.S. = R.H.S.

if $\cos 2\theta = 1$ and $\sin 2\theta = 0$

$2\theta = 2n\pi$ and $2\theta = n\pi$

$\theta = n\pi$ and $\theta = \frac{n\pi}{2}$

$\theta = -2\pi, -\pi, 0, \pi, 2\pi$

number of value of $\theta = 5$

21.26 (3)

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 5 & 7 \\ 5 & 8 & 8 \end{bmatrix}$$

$|A| = 2(40 - 56) + 3(24 - 35) + 1\,(24 - 25)$
$= -32 - 33 - 1$
$= -66$
$|A| \neq 0$
so rank of A = 3 = α

$$\int_0^{\sqrt{3}} [x^2]\,dx = \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx = 0 + \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2}$$

$$= 2\sqrt{3} - \sqrt{2} - 1 = \sqrt{12} - \sqrt{2} - 1$$

21.27 (3)

$f(\sin x) = \sin x\,(\sin^2 x - \cos^2 x) - 2\cos x(\sin x - \sin x) + 2\tan x\,(\cos x - \sin x \tan x)$
$= \sin x \,.\, \cos 2x \,.\, (2\sec^2 x - 1)$

$$f(\sin x) = \sin x\,(1 - 2\sin^2 x)\left(\frac{2-\cos^2 x}{\cos^2 x}\right) = \sin x\,(1 - 2\sin^2 x)\left(\frac{1+\sin^2 x}{1-\sin^2 x}\right)$$

$$f(x) = \frac{x(1-2x^2)(1+x^2)}{(1-x^2)}$$

$\lim\limits_{x\to 1} f(x)$ = not define (obvious)

and $g(x) = \int f(x)dx = \frac{x^4}{2} + \frac{3x^2}{2} + \ell n\,(x^2 - 1) + c$

$g(\sqrt{2}) = 0 \quad \Rightarrow \quad c = -5$

$$g(x) = \frac{x^4}{2} + \frac{3x^2}{2} + \ell n\,(x^2 - 1) - 5$$

$g(2) = 9 + \ell n\,3$
$g(-x) = g(x) \quad \Rightarrow$ even function
Time period of sin x is 2π and $\sin^2 x$, $\cos^2 x$ is π
So L.C.M. of 2π and π is 2π
so time period of f(sinx) is 2π.

21.28 (3)

$\Delta(x)$ is continuous function in $\left[0, \frac{\pi}{2}\right]$ and $\Delta\left(\frac{\pi}{6}\right) = \Delta\left(\frac{\pi}{4}\right) = 0$ and it is differentiable in $\left(0, \frac{\pi}{2}\right)$

$\Delta(0) = \sqrt{6} - 2$

so by Rolle's theorem $\Delta'(x) = 0$

have atleast one solution in $\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$

21.29 **(1)**

$$D = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = \lambda^3 - 3\lambda + 2 = (\lambda - 1)^2 (\lambda + 2)$$

at $\lambda = 1$, $D = D_1 = D_2 = D_3 = 0$ so system have infinite solution. So system is consistent
at $\lambda = -2$, $D = 0$ but $D_1 \neq 0$ so system have no solution. So system is inconsistent

$$\lim_{x \to -2} \frac{|x+2|}{(x+2)(x-2)}$$

L.H.L. = $\frac{1}{4}$, R.H.L. = $-\frac{1}{4}$

L.H.L. ≠ R.H.L.
so limit does not exist.

21.30 **(1)**

$$\Delta = -(\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma) = -(\alpha + \beta + \gamma)\,[(\alpha + \beta + \gamma)^2 - 3(\alpha\beta + \beta\gamma + \gamma\alpha)] = \left(\frac{b}{a}\right)\left(\frac{b^2}{a^2} - \frac{3c}{a}\right] = \left(\frac{b^3 - 3abc}{a^3}\right)$$

21.31 **(4)**

Statement-1 : $\Delta = i^{99} \times i^{98} \times 2i^{97} = 2 \times i^{294} = 2 \times i^2 = -2$
so first statement is true.

Statement-2 : It is a skew-symmetric determinant which is zero only for odd order determinant.
so second statement is false.

21.32 **(4)**

For a unique solution of system of equation AX = B. A should be non-singular but it does not depend on B
Statement-1 is false.
Statement-2 is correct.

21.33 **(4)**

'A' is singular matrix So A^{-1} is not possible and a + b + c = 1 is satisfied by a = 1, b = 0, c = 0 which makes the B as a unit matrix and explain AB = BA but for other set of values of (a, b, c) it is not true so not always correct. So statement-1 is not true, statement-2 is always true.

21.34 **(1)**

$$\Delta = \begin{vmatrix} 1 & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & 1 & \frac{\log z}{\log x} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & 1 \end{vmatrix}$$

$$\Delta = \frac{1}{\log x \log y \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} \Rightarrow \quad \Delta = 0$$

21.35 **(1)**

A = cafactor matrix of B

$\therefore \quad |A| = |B|^2$

22. Complex Number

22.1 (2)

$\frac{z+1}{z+i}$ is purely Imaginary

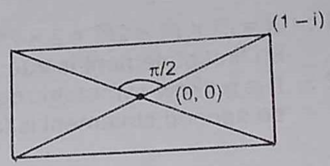
So $\frac{z+1}{z+i} + \frac{\bar{z}+1}{\bar{z}-i} = 0$

$\Rightarrow \quad z\bar{z} + z\left(\frac{1-i}{2}\right) + \bar{z}\left(\frac{1+i}{2}\right) = 0$

radius = $\sqrt{\left(\frac{1}{4}+\frac{1}{4}\right) - 0} = \frac{1}{\sqrt{2}}$

22.2 (1)

Vertices are $(1-i)e^{\pm i\pi/2}$



$(1-i)(\pm i)$

So vertices ax $1 + i$ & $-1 - i$

22.3 (3)

$(x-y) + i(3x-y) = \frac{i^3 + (-1)i^{-3}}{(-i)^{-1} - i^2} = -1 - i$

by comparision

$x - y = -1$

$3x - y = -1$

So $x = 0, y = 1$

22.4 (2)

$x^7 - 1 = (x-1)(x-\alpha_1)(x-\alpha_2) \,......... (x-\alpha_6)$

put x = 3

$3^7 - 1 = 2(3-\alpha_1)(3-\alpha_2) \,.......... (3-\alpha_6)$

but $|3-\alpha_1| = |3-\alpha_6|$

$|3-\alpha_2| = |3-\alpha_5|$

$|3-\alpha_3| = |3-\alpha_4|$

So $|(3-\alpha_1)(3-\alpha_3)(3-\alpha_5)| = \sqrt{\frac{3^7-1}{2}} = \sqrt{1093}$

22.5 **(4)**

$x^n - 1 = (x-1)(x-\alpha_1)(x-\alpha_2) \dots\dots\dots (x-\alpha_{n-1})$

$\log(x^n-1) = \log(x-1) + \log(x-\alpha_1) + \dots\dots\dots + \log(x-\alpha_{n-1})$

Differentiate w.r.t. x

$$\frac{nx^{n-1}}{x^n-1} = \frac{1}{x-1} + \frac{1}{x-\alpha_1} + \dots\dots\dots\dots + \frac{1}{x-\alpha_{n-1}}$$

put x = 2

$$\frac{1}{2-\alpha_1} + \frac{1}{2-\alpha_2} + \dots\dots\dots + \frac{1}{2-\alpha_{n-1}} = \frac{n2^{n-1}}{2^n-1} - 1$$

$$= \frac{(n-2).2^{n-1}+1}{2^n-1}$$

22.6 **(3)**

$$(x-y) + i(3x-y) = \frac{i^3 + (-1)i^{-3}}{(-i)^{-1} - i^2}$$

$(x-y) + i(3x-y) = -1 - i$

by comparison

$x - y = -1$

$3x - y = -1$

$x = 0,\ y = 1$

22.7 **(2)**

$$Z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$$

$$= \frac{(3-4\sin^2\theta) + i(6\sin\theta + 2\sin\theta)}{1+4\sin^2\theta}$$

Z is purely real if imaginary part is zero

so $$\frac{8\sin\theta}{1+4\sin^2\theta} = 0$$

$\Rightarrow$ $\sin\theta = 0$

$\theta = n\pi$

22.8 **(3)**

$$\text{Sin}\ \frac{\pi}{5} + i\left(1-\cos\frac{\pi}{5}\right) = 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + 2i\sin^2\frac{\pi}{10}$$

$$= 2\sin\frac{\pi}{10}\left[\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right]$$

so amplitude $= \frac{\pi}{10}$

22.9 (2)

$\sqrt{-7-24i} = a + ib$

by squaring both side

$(a^2 - b^2) + (2a\,b)i = -7 - 24\,i$

$a^2 - b^2 = -7$ .... (1)

$2ab = -24$ .... (2)

$(1)^2 + (2)^2$

$(a^2 - b^2)^2 + (2ab)^2 = 49 + 576$

$(a^2 + b^2)^2 = 625$

$\Rightarrow a^2 + b^2 = 25$ ...(3)

by adding (1) & (3)

$2a^2 = 18 \Rightarrow a = \pm 3$

& $b = \pm 4$

from (2) product of ab is negative so

$a = +3$ & $b = -4$

or $a = -3$ & $b = 4$

so $a^3 + b^3 = 27 - 64 = -37$

or $a^3 + b^3 = -27 + 64 = 37$

22.10 (4)

$||z| - 4| \le |z - 4| \le |z| + 4$

$|2 - 4| \le |z - 4| \le 2 + 4$

$2 \le |z - 4| \le 6$

22.11 (2)

$|3z - 2| = |3z - 4|$

$\Rightarrow \left|z - \frac{2}{3}\right| = \left|z - \frac{4}{3}\right|$

It represent straight line

22.12 (4)

$$z_2 = z_1 e^{i(2\pi/3)} = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$= \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$= -1$

22.13 (1)

$\overline{(x+iy)(1-2i)} = 1 + i$

$(x - iy)(1 + 2i) = 1 + i$

$\Rightarrow (x + 2y) + i(2x - y) = 1 + i$

so $x + 2y = 1$

$2x - y = 1$

$x = \frac{3}{5}$, $y = \frac{1}{5}$

22.14 (1)

$z^n = (z + 1)^n$

$|z|^n = |z + 1|^n$

$\Rightarrow |z| = |z + 1|$

It represent prependicular bisector of line joining (0, 0) and (–1, 0) so its equation is $x = -\frac{1}{2}$

$\Rightarrow 2x + 1 = 0$

22.15 **(2)**

$z^3 + 2z^2 + 2z + 1 = 0$

$(z + 1)(z^2 + z + 1) = 0$

$z = -1, \omega, \omega^2$

only ω and ω^2 satisfy Ist equation

so sum of roots $\omega + \omega^2 = -1$

22.16 **(2)**

$$1 + 2x + 3x^2 + \ldots\ldots 3n.\ x^{3n-1} = \frac{d}{dx}[x + x^2 + \ldots\ldots + x^{3n}]$$

$$= \frac{d}{dx}\left[\frac{x - x^{3n+1}}{1-x}\right]$$

$$= \frac{(1-x)[1-(3n+1).x^{3n})] + (x - x^{3n+1})}{(1-x)^2}$$

put $x = \omega$

$$1 + 2\omega + 3\omega^2 + \ldots\ldots + 3n\omega^{3n-1} = \frac{(1-\omega)(1-3n-1)+(\omega-\omega)}{(1-\omega)^2}$$

$$= -\frac{3n}{1-\omega}$$

$$= \frac{3n(\omega^2-1)}{(\omega-1)(\omega^2-1)}$$

$$= 3n(\omega^2 - 1)$$

22.17 **(2)**

$x + y + z = a(1 + \omega + \omega^2) + b(1 + \omega + \omega^2) = 0$

so $\quad x^3 + y^3 + z^3 = 3xyz$

$\quad = 3(a^3 + b^3) \qquad$ (because $xyz = a^3 + b^3$)

22.18 **(3)**

$$\frac{z^{2n}-1}{z^{2n}+1} = \frac{(\cos\theta + i\sin\theta)^{2n} - 1}{(\cos\theta + i\sin\theta)^{2n} + 1}$$

$$= \frac{\cos 2n\theta + i\sin 2n\theta - 1}{\cos 2n\theta + i\ \sin 2n\theta + 1}$$

$$= \frac{-2\sin^2 n\theta + 2i\sin n\theta.\cos n\theta}{2\cos^2 n\theta + i2\sin n\theta\cos n\theta}$$

$$= \frac{i2\sin n\theta[\cos n\theta + i\sin n\theta]}{2\cos n\theta[\cos n\theta + i\sin n\theta]}$$

$= i\tan n\theta$

22.19 **(3)**

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

$(z_1 + z_2)(\bar{z}_1 + \bar{z}_2) = z_1\bar{z}_1 + z_2\bar{z}_2$

$\Rightarrow \qquad z_1\bar{z}_2 + z_2\bar{z}_1 = 0$

It means $z_1\bar{z}_2$ is imaginary

Now $\quad \dfrac{z_1}{z_2} = \dfrac{z_1\bar{z}_2}{z_2\bar{z}_2} = \dfrac{\text{Imaginary}}{\text{Real}} = \text{Imaginary}$

22.20 (1)

Compare with $z\bar{z} + \bar{a}z + a\bar{z} + b = 0$

$a = 2 + 3i,\ b = 4$

$\bar{a} = 2 - 3i$

so radius = $\sqrt{a\bar{a} - b} = \sqrt{(2+3i)(2-3i) - 4} = 3$

22.21 (2)

$z = r(\cos\theta + i\sin\theta)$

$e^{iz} = e^{r(-\sin\theta + i\cos\theta)} = e^{-r\sin\theta}.e^{i(r\cos\theta)}$

so $\arg(e^{iz}) = r\cos\theta$

22.22 (2)

$x^7 - 1 = (x - 1)(x - \alpha_1)(x - \alpha_2)\ ....(x - \alpha_6)$

put $x = 3$

$3^7 - 1 = 2(3 - \alpha_1)(3 - \alpha_2)\ ....(3 - \alpha_6)$

but $|3 - \alpha_1| = |3 - \alpha_6|$

$|3 - \alpha_2| = |3 - \alpha_5|$

$|3 - \alpha_3| = |3 - \alpha_4|$

so $|(3 - \alpha_1)(3 - \alpha_3)(3 - \alpha_5)| = \sqrt{\dfrac{3^7 - 1}{2}} = \sqrt{1093}$

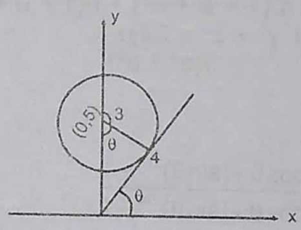
22.23 (4)

From figure

$z = 4\cos\theta + i\,4\sin\theta$

$= 4 \cdot \dfrac{3}{5} + i\,4 \cdot \dfrac{4}{5}$

$= \dfrac{12 + 16i}{5}$



22.24 (2)

$$\frac{z - (1+i)}{z + (1+i)} + \frac{\bar{z} - (1-i)}{\bar{z} + (1-i)} = 0$$

$\Rightarrow\ (z\bar{z} + z(1-i) - \bar{z}(1+i) - 2) + (z\bar{z} - z(1-i) + \bar{z}(1+i) - 2) = 0$

$\Rightarrow\ z\bar{z} = 2$

locus of z is circle.

22.25 (2)

$z = 3 - 4i$

Newly obtained vector is $= 2.5\,(3 - 4i)e^{i\pi}$

$= \dfrac{5}{2}(3 - 4i)(-1 + 0i)$

$= -\dfrac{15}{2} + 10\,i$

22.26 **(2)**

$2z^2 + 2z + \lambda = 0$

$z = \dfrac{-2 \pm \sqrt{4-8\lambda}}{4}$

for non-real roots $4 - 8\lambda < 0$

$\lambda > \dfrac{1}{2}$

for equilateral triangle

$|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$

$\Rightarrow \quad \left|0 - \dfrac{-1+i\sqrt{2\lambda-1}}{2}\right| = \left|\dfrac{-1+i\sqrt{2\lambda-1}}{2} - \dfrac{-1-i\sqrt{2\lambda-1}}{2}\right| = \left|0 - \dfrac{-1-i\sqrt{2\lambda-1}}{2}\right|$

or $\quad \dfrac{1}{4}(1 + 2\lambda - 1) = (\sqrt{2\lambda-1})^2$

$\Rightarrow \quad \dfrac{\lambda}{2} = 2\lambda - 1$

$\Rightarrow \quad \lambda = \dfrac{2}{3}$

22.27 **(4)**

$x^n - 1 = (x-1)(x-\alpha_1)(x-\alpha_2) \ldots\ldots (x-\alpha_{n-1})$

$\log(x^n - 1) = \log(x-1) + \log(x-\alpha_1) + \log(x-\alpha_2) + \ldots\ldots + \log(x-\alpha_{n-1})$

Differentiate w.r.t x

$\dfrac{nx^{n-1}}{x^n-1} = \dfrac{1}{x-1} + \dfrac{1}{x-\alpha_1} + \dfrac{1}{x-\alpha_2} + \ldots.. + \dfrac{1}{x-\alpha_{n-1}}$

put x = 2

$\dfrac{1}{2-\alpha_1} + \dfrac{1}{2-\alpha_2} + \ldots. + \dfrac{1}{2-\alpha_{n-1}} = \dfrac{n.2^{n-1}}{2^n-1} - 1 = \dfrac{(n-2)2^{n-1}+1}{2^n-1}$

22.28 **(4)**

$iz^3 + z^2 - z + i = 0$

$z^3 + \dfrac{1}{i}z^2 - \dfrac{1}{i}z + 1 = 0$

$z^3 - iz^2 + iz + 1 = 0$

$z^2(z-i) + i(z-i) = 0$

$(z-i)(z^2+i) = 0$

$z = i$ or $z^2 = -i$

$|z| = |i|$ or $|z|^2 = |-i|$

$\Rightarrow \quad |z| = 1$

or $\quad |z|^2 = 1$

so $\quad |z| = 1$

22.29 **(4)**

$(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n$

$= \left(2\cos^2\dfrac{\theta}{2} + 2i\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\right)^n + \left(2\cos^2\dfrac{\theta}{2} - 2i\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\right)^n$

$= 2^n\cos^n\dfrac{\theta}{2}\left[\cos\dfrac{n\theta}{2} + i\sin\dfrac{n\theta}{2} + \cos\dfrac{n\theta}{2} - i\sin\dfrac{n\theta}{2}\right]$

$= 2^{n+1} . \cos^n\dfrac{\theta}{2} . \cos\dfrac{n\theta}{2}$

22.30 **(2)**

$$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$$

$$=\frac{\omega(a+b\omega+c\omega^2)}{b\omega+c\omega^2+a\omega^3}+\frac{\omega^2(a+b\omega+c\omega^2)}{c\omega^2+a\omega^3+b\omega^4}$$

$= \omega + \omega^2 = -1$

22.31 **(3)**

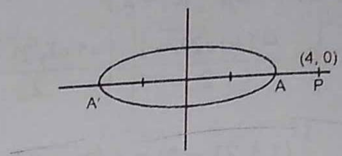
$|z-1| + |z+3| \le 8$

$A'(-5, 0)\ A(3, 0)$

so minimum $|z-4| = AP = 1$

maximum $|z-4| = A'P = 9$

so $|z-4| \in [1, 9]$



22.32 **(2)**

$-3 - x^2yi = x^2 + y + 4i$

so $x^2 + y = -3 \Rightarrow y = -3 - x^2$

$x^2y = -4$

$\Rightarrow x^2(3 + x^2) = 4$

$x^4 + 3x^2 - 4 = 0$

$$x^2=\frac{-3\pm\sqrt{9+16}}{2}=\frac{-3\pm 5}{2}$$

$x^2 = -4, 1$

$x^2 = 1$ (x is real)

$|x| = 1$

$y = -4\ |y| = 4$

$|x| + |y| = 5$

22.33 **(2)**

$(x^n - 1) = (x-1)(x-\alpha)(x-\alpha^2) \ldots\ldots (x-\alpha^{n-1})$

$$\Rightarrow (x-\alpha)(x-\alpha^2)\ldots\ldots(x-\alpha^{n-1})=\frac{x^n-1}{x-1}$$

$= 1 + x + \ldots.. + x^{n-1}$

Put x = 1

$(1-\alpha)(1-\alpha^2)\ldots\ldots(1-\alpha^{n-1}) = n$

22.34 **(1)**

$$\left|\frac{3z+i}{2z+3+4i}\right|=\frac{3}{2}$$

$$\frac{3\left|z+\frac{i}{3}\right|}{2\left|z+\frac{3+4i}{2}\right|}=\frac{3}{2}$$

$$\Rightarrow \left|z+\frac{i}{3}\right|=\left|z+\frac{3+4i}{2}\right|$$

it is equation of perpendicular bisector

22.35 (4)

$$(f(\theta))^2 = \frac{4}{(3+\sin\theta)^2+\cos^2\theta} = \frac{4}{10+6\sin\theta}$$

$$= \frac{2}{5+3\sin\theta}$$

so $\frac{1}{4} \le (f(\theta))^2 \le 1$

$\Rightarrow \quad \frac{1}{2} \le f(\theta) \le 1$

22.36 (2)

$$\frac{|z+1|}{|z-1|} = 2$$

Diametrically opposite ends are $\left(\frac{1}{3}, 0\right)$ and (3, 0)

centre $\left(\frac{5}{3}, 0\right)$

radius = $\frac{4}{3}$

22.37 (1)

Both are currect, St.1 is correct explanation of St.2

$0 \le |z_1 + z_2 + z_3|^2$

$\le |z_1|^2 + |z_2|^2 + |z_3|^2 + 2\text{ Re }(z_2\bar{z}_3 + z_3\bar{z}_1 + z_1\bar{z}_2)$

$\Rightarrow \quad \text{Re }(z_2\bar{z}_3 + z_3\bar{z}_1 + z_1\bar{z}_2) \ge -\frac{3}{2}$

next, $|z_2 - z_3|^2 + |z_3 - z_1|^2 + |z_1 - z_2|^2$

$= 2(|z_1|^2 + |z_2|^2 + |z_3|^2) - 2\text{ Re}(z_2\bar{z}_3 + z_3\bar{z}_1 + z_1\bar{z}_2)$

$\le \quad 2(1 + 1 + 1) + 2\left(\frac{3}{2}\right) = 9$

22.38 (3)

$$= \left|\frac{2+3\omega+4z\omega^2}{4\omega+3\omega^2 z+2z}\right| = \left|\frac{1}{z}\frac{2+3\omega+4z\omega^2 z}{2+3\omega^2+4\omega z^{-1}}\right|$$

$$= \left|\frac{1}{2}\right|\left|\frac{2+3\omega+4\omega^2 z}{2+3\bar{\omega}+4\bar{\omega}^2\bar{z}}\right| = \frac{1}{|z|} = 1$$

$$\left[\because \bar{\omega} = \omega^2, \bar{z} = \frac{1}{z}\right]$$

statement 2 is false

23. Vector

23.1 **(2)**
Equation of bisector will be of the form :

$t\left(\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}}{|\vec{b}|}\right)$, where t : real parameter

i.e. $t\left(\frac{14}{15}(\hat{i}+\hat{j}-\hat{k})\right)$

Now $\sqrt{3\left(\frac{14}{15}\right)^2 t^2} = \frac{7}{3}$ $\therefore$ $t = \pm\frac{5}{2\sqrt{3}}$

$\therefore$ Required vector $= \pm\frac{5}{2\sqrt{3}}\left(\frac{14}{15}\hat{i}+\frac{14}{15}\hat{j}-\frac{14}{15}\hat{k}\right)$

$= \pm\frac{7}{3\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$

23.2 **(4)**
Let $\vec{a} = 6\hat{i}+6\hat{k}$

$\vec{b} = 4\hat{j}+2\hat{k}$

$\vec{c} = 4\hat{j}-8\hat{k}$

then $\vec{a}\times\vec{b} = 12(-2\hat{i}-\hat{j}+2\hat{k})$

$\therefore$ area of base of the

parallelopiped $= \frac{1}{2}|\vec{a}\times\vec{b}| = 18$

Height of parallelopiped

= length of projection of $\vec{c}$ on $\vec{a}\times\vec{b}$

$= \frac{|\vec{c}.(\vec{a}\times\vec{b})|}{|\vec{a}\times\vec{b}|} = \frac{20}{3}$

$\therefore$ Volume of parallelopiped

$= 18 \times \frac{20}{3} = 120$

23.3 **(1)**

$(\vec{a}-\vec{b})\times[(\vec{b}+\vec{a})\times(2\vec{a}+\vec{b})] = \vec{b}+\vec{a}$

$\Rightarrow$ $\{(\vec{a}-\vec{b}).(2\vec{a}+\vec{b})\}(\vec{b}+\vec{a}) - \{(\vec{a}-\vec{b}).(\vec{b}+\vec{a})\}(2\vec{a}+\vec{b}) = \vec{b}+\vec{a}$

$\Rightarrow$ $(2-\vec{a}.\vec{b}-1)(\vec{b}+\vec{a}) = \vec{b}+\vec{a}$

$\Rightarrow$ either $\vec{b}+\vec{a} = \vec{0}$ or $1-\vec{a}.\vec{b} = 1$

$\Rightarrow$ either $\vec{b} = -\vec{a}$ or $\vec{a}.\vec{b} = 0 \Rightarrow$ either $\theta = \pi$ or $\theta = \frac{\pi}{2}$

23.4 (2)

Since given line and plane are parallel as $(\hat{i}-\hat{j}+4\hat{k}).(\hat{i}+5\hat{j}+\hat{k})=0$

$\therefore$ Distance between any point on given line and given plane gives required result

$\therefore$ Required distance = $\left|\dfrac{(2\hat{i}-2\hat{j}+3\hat{k}).(\hat{i}+5\hat{j}+\hat{k})-5}{\sqrt{1+25+1}}\right| = \dfrac{10}{3\sqrt{3}}$ unit

23.5 (1)

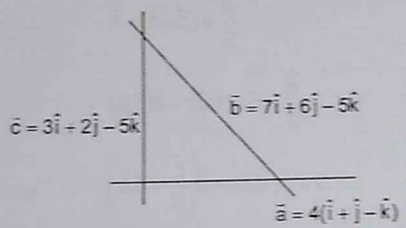
$V_2 = \dfrac{1}{6}\left|[\vec{p}\,\vec{q}\,\vec{r}]\right|$

$V_2 = \dfrac{1}{6}\left\|\begin{matrix}1 & 1 & -2\\ 3 & -2 & 1\\ 1 & -4 & 2\end{matrix}\right\|\left|[\vec{a}\,\vec{b}\,\vec{c}]\right|$

$V_2 = \dfrac{1}{6}\ 15\ V_1$

$\therefore \quad \dfrac{V_2}{V_1} = \dfrac{5}{2}$

23.6 (1)



$\vec{a}.\vec{c} = 4(3+2-5)=0$

$\vec{a}\perp\vec{c}$

Right angled Δ.

23.7 (2)

$\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{(x\hat{i}-3\hat{j}+\hat{k}).(x\hat{i}-3x\hat{j}+2\hat{k})}{|\vec{a}||\vec{b}|} < 0$

$x^2+3x+2<0$

$(x+2)(x+1)<0$

$x \in (-2, -1)$

no integral values of x

23.8 (1)

$\vec{A}.\vec{B} = 0 \quad \Rightarrow \quad \vec{A}\perp\vec{B}$

$\vec{A}.\vec{C} = 0 \quad \Rightarrow \quad \vec{A}\perp\vec{C}$

$\therefore \quad \vec{A}\,\|\,\vec{B}\times\vec{C}\ , \quad \vec{A}=k(\vec{B}\times\vec{C})$

$|\vec{A}| = |k\ \vec{B}\times\vec{C}| = |k|\ |\vec{B}||\vec{C}|\ \sin\dfrac{\pi}{6}$

$1 = |k|\ .\ 1\ .\ 1.\dfrac{1}{2}$

$|k| = 2 \Rightarrow \quad k = \pm 2$

23.9 (1)

$\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$

Now $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{vmatrix}$$

or $(y-z)\hat{i} + (z-x)\hat{j} + (x-y)\hat{k} = -10\hat{i} + 3\hat{j} + 7\hat{k}$

$y - z = -10$; $z - x = 3$; $x - y = 7$

$\vec{R}.\vec{A} = 0 \quad \Rightarrow \quad 2x + z = 0$

after solving $x = -1, y = -8$

$z = 2$

$\vec{R} = -\hat{i} - 8\hat{j} + 2\hat{k}$

23.10 (1)

$\vec{a} \times \vec{b} = \vec{c} \quad \Rightarrow \quad (\vec{a} \times \vec{b}).\vec{c} = \vec{c}.\vec{c}$

$\therefore \quad [\vec{a}\,\vec{b}\,\vec{c}] = |\vec{c}|^2$

$\vec{b} \times \vec{c} = \vec{a} \quad \Rightarrow \quad (\vec{b} \times \vec{c}).\vec{a} = \vec{a}.\vec{a}$

$\therefore \quad [\vec{b}\,\vec{c}\,\vec{a}] = |\vec{a}|^2$

$\vec{c} \times \vec{a} = \vec{b} \quad \Rightarrow \quad (\vec{c} \times \vec{a}).\vec{b} = \vec{b}.\vec{b}$

$\therefore \quad [\vec{c}\,\vec{a}\,\vec{b}] = |\vec{b}|^2$

But $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$

$\therefore \quad |\vec{c}|^2 = |\vec{a}|^2 = |\vec{b}|^2$

$\therefore \quad |\vec{a}| = |\vec{b}| = |\vec{c}|$

23.11 (1)

Vector along the angle bisector

$= \hat{a} + \hat{b}$

$$= \left(\frac{-4\hat{i} + 3\hat{k}}{5}\right) + \left(\frac{14\hat{i} + 2\hat{j} - 5\hat{k}}{15}\right)$$

$$= \frac{2(\hat{i} + \hat{j} + 2\hat{k})}{15}$$

$\therefore \quad \vec{d} = (\hat{i} + \hat{j} + 2\hat{k})$

23.12 (4)

In ΔAPC $\quad \overrightarrow{PA} + \overrightarrow{AC} + \overrightarrow{CP} = 0 \quad$ ......(1)

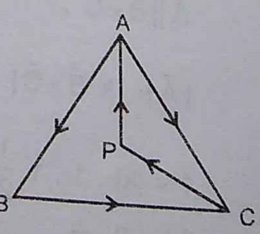
In ΔABC

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

from equation (1)

$\overrightarrow{PA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CP} = 0$

$\Rightarrow \quad \overrightarrow{PA} + \overrightarrow{CP} = \overrightarrow{BA} + \overrightarrow{CB}$

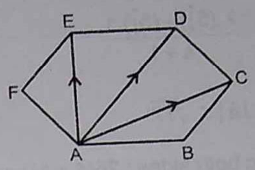


23.13 (2)

By triangle law $\overrightarrow{AB} = \overrightarrow{AD} - \overrightarrow{BD}, \overrightarrow{AC} = \overrightarrow{AD} - \overrightarrow{CD}$

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$$

$$= 3\overrightarrow{AD} + (\overrightarrow{AE} - \overrightarrow{BD}) + (\overrightarrow{AF} - \overrightarrow{CD})$$



$= 3\overrightarrow{AD}$

$\because$ $(\overrightarrow{AE} = \overrightarrow{BD}; \overrightarrow{AF} = \overrightarrow{CD})$

$\lambda = 3$

23.14 (1)

$\vec{a}.\vec{c} = 1$ and $\vec{b}.\vec{c} = 1$

Given that, $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{c}.\vec{a})\vec{b} - (\vec{c}.\vec{b})\vec{a} = \mu\vec{b} + \lambda\vec{a}$

$\mu = c.a = 1$; $\lambda = -(c.b) = -1$

$\Rightarrow$ $\lambda + \mu = 1 - 1 = 0$

23.15 (4)

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

$\therefore$ $\vec{a} \times (\vec{b} \times \vec{c}) = \dfrac{\vec{b}}{2}$

$(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b}) \times \vec{c} = \dfrac{\vec{b}}{2}$

or $\left(\vec{a}.\vec{c} - \dfrac{1}{2}\right)\vec{b} - (\vec{a}.\vec{b})\vec{c} = 0$

$\vec{a}.\vec{c} - \dfrac{1}{2} = 0$; $a.b = 0$ $\Rightarrow$ $\alpha = 90°$

$\Rightarrow$ $|a|\ |c| \cos\theta = \dfrac{1}{2}$ $\Rightarrow$ $\cos\theta = \dfrac{1}{2}$

$\Rightarrow$ $\theta = 60°$

23.16 (2) $(\vec{r} - \vec{q}) \times \vec{p} = 0$

$\Rightarrow$ $\vec{r} - \vec{q} = \lambda\vec{p}$, $\vec{r} = \lambda\vec{p} + \vec{q}$ — (1) now $\vec{r} . \vec{s} = 0$

$\Rightarrow$ $(\lambda\vec{p} + \vec{q}) . \vec{s} = 0$ $\Rightarrow$ $\lambda = -\dfrac{\vec{q} . \vec{s}}{\vec{p} . \vec{s}}$ — (2) Put λ from (2) in (1) to get the result]

23.17 (1)

$$\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]};\ \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]};\ \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

Substitute the values of $\vec{p}, \vec{q}, \vec{r}$ to get the result

23.18 (3)

The point that divides $5\hat{i}$ and $5\hat{j}$ in the ratio of $\lambda : 1$ is

$$\Rightarrow \quad \frac{\lambda(5\hat{j}) + (5\hat{i}).1}{\lambda+1} \qquad \therefore \quad \vec{a} = \frac{5\hat{i} + 5\lambda\hat{j}}{\lambda+1}$$

also $|\vec{a}| \le \sqrt{17}$ $\Rightarrow$ $\frac{1}{|\lambda+1|}\sqrt{25+25\lambda^2} \le \sqrt{17}$

Squaring both sides : $25(1+\lambda^2) \le 17(\lambda^2 + 2\lambda + 1)$

$8\lambda^2 - 34\lambda + 8 \le 0$

$\Rightarrow \quad 4\lambda^2 - 17\lambda + 4 \le 0 \quad \Rightarrow \quad (\lambda - 4)(4\lambda - 1) \le 0$

$\lambda \in \left[\frac{1}{4}, 4\right]$

23.19 (2)

$L_1 \quad \vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$

$L_2 \quad \vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$

point $(\hat{i} + \hat{j})$ lies in plane

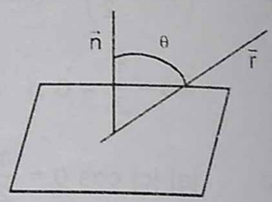
normal $= (\hat{i} + 2\hat{j} - \hat{k}) \times (-\hat{i} + \hat{j} - 2\hat{k})$

$= (-3\hat{i} + 3\hat{j} + 3\hat{k})$

plane $(\vec{r} - \hat{i} - \hat{j}).(-\hat{i} + \hat{j} + \hat{k}) = 0$

$\Rightarrow \quad \vec{r}.(\hat{i} - \hat{j} - \hat{k}) = 0$

23.20 (2)



$$\cos\theta = \frac{(\hat{i} - \hat{j} + \hat{k}).(2\hat{i} - \hat{j} + \hat{k})}{\sqrt{3}\sqrt{6}} = \frac{2+1+1}{\sqrt{3}\sqrt{6}}$$

$$\cos\theta = \frac{2\sqrt{2}}{3}$$

23.21 (2)

$\vec{n}_1 \times \vec{n}_2 = (\hat{i} - 3\hat{j} + \hat{k}) \times (2\hat{i} + 5\hat{j} - 3\hat{k})$

$= 4\hat{i} + 5\hat{j} + 11\hat{k}$

23.22 (1)

$$\frac{(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2}{2\vec{a}^2\vec{b}^2} = \frac{\vec{a}^2\vec{b}^2(\sin^2\theta + \cos^2\theta)}{2\vec{a}^2\vec{b}^2} = \frac{1}{2}$$

23.23 (3)

Let $\alpha \ne 0$, then $\alpha(\vec{a} \times \vec{b}).\vec{c} + \beta(\vec{b} \times \vec{c}).\vec{c} + \gamma(\vec{c} \times \vec{a}).\vec{c} = 0$

$\Rightarrow \quad \alpha[\vec{a}\ \vec{b}\ \vec{c}] = 0 \quad \Rightarrow \quad [\vec{a}\ \vec{b}\ \vec{c}] = 0 \qquad (\because \alpha \ne 0)$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

23.24 (1)

$$\frac{\vec{a}.\vec{b}\times\vec{c}}{\vec{c}\times\vec{a}.\vec{b}}+\frac{\vec{b}.\vec{a}\times\vec{c}}{\vec{c}.\vec{a}\times\vec{b}}=\frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{c}\vec{a}\vec{b}]}+\frac{[\vec{b}\vec{a}\vec{c}]}{[\vec{c}\vec{a}\vec{b}]}$$

$$=\frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{c}\vec{a}\vec{b}]}-\frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{c}\vec{a}\vec{b}]}=0$$

23.25 (1)

Since $\vec{a},\vec{b}$ and $\vec{a}\times\vec{b}$ are non - coplanar, hence

$\vec{r}=x\vec{a}+y\vec{b}+z(\vec{a}\times\vec{b})$

$\vec{b}=\vec{r}\times\vec{a}=(x\vec{a}+y\vec{b}+z(\vec{a}\times\vec{b}))\}\times\vec{a}$

$=y(\vec{b}\times\vec{a})+z[(\vec{a}\times\vec{b})\times\vec{a}]=-y(\vec{a}\times\vec{b})-z[\vec{a}\times(\vec{a}\times\vec{b})]$

$=-y(\vec{a}\times\vec{b})-z[(\vec{a}.\vec{b})\vec{a}-(\vec{a}.\vec{a})\vec{b}]$

$=-y(\vec{a}\times\vec{b})+z[(\vec{a}.\vec{a})b$

$y=0,\quad z=\frac{1}{\vec{a}.\vec{a}}\qquad\Rightarrow\qquad \vec{r}=x\vec{a}+\frac{1}{\vec{a}.\vec{a}}(\vec{a}\times\vec{b})$

23.26 (2)

Let the foot of perpendicular be $(4\lambda, 3\lambda-3, 5\lambda-10)$

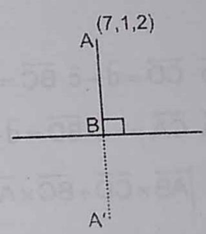
so direction ratios of perpendicular line are $<4\lambda-7, 3\lambda-4, 5\lambda-12>$

Also $4(4\lambda-7)+3(3\lambda-4)+5(5\lambda-12)=0$

$\Rightarrow\quad \lambda=2$

then foot of perpendicular is B(8, 3, 0)

so position vector of image is $A'(9\hat{i}+5\hat{j}-2\hat{k})$



23.27 (1)

$\vec{r}=\lambda_1\vec{r}_1+\lambda_2\vec{r}_2+\lambda_3\vec{r}_3$

$\Rightarrow\quad 2\vec{a}-3\vec{b}+4\vec{c}=(\lambda_1-\lambda_2+\lambda_3)\vec{a}+(-\lambda_1+\lambda_2+\lambda_3)\vec{b}+(\lambda_1+\lambda_2+\lambda_3)\vec{c}$

$\lambda_1-\lambda_2+\lambda_3=2\;|-\lambda_1+\lambda_2+\lambda_3=-3|\;\lambda_1+\lambda_2+\lambda_3=4$

$\Rightarrow\quad \lambda_1=\frac{7}{2},\ \lambda_2=1,\ \lambda_3=-\frac{1}{2}$

23.28 (3)

Required unit vector $=\pm\frac{\vec{a}\times(\vec{a}\times\vec{b})}{|\vec{a}\times(\vec{a}\times\vec{b})|}$

Now $\quad\vec{a}\times(\vec{a}\times\vec{b})=(\vec{a}.\vec{b})\vec{a}-(\vec{a}.\vec{a})\vec{b}$

$\vec{a}\times\vec{b}=-2\hat{j}-2\hat{k}$

$\vec{a}\times(\vec{a}\times\vec{b})=-4\hat{i}+2\hat{j}-2\hat{k}$

$|\vec{a}\times(\vec{a}\times\vec{b})|=2\sqrt{6}$

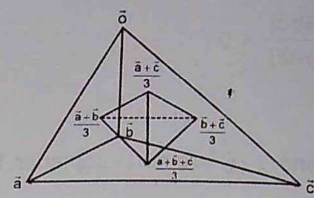
unit vector $=\pm\frac{\vec{a}\times(\vec{a}\times\vec{b})}{|\vec{a}\times(\vec{a}\times\vec{b})|}=\pm\frac{(2\hat{i}-\hat{j}+\hat{k})}{\sqrt{6}}$

23.29 **(3)**

Let the given tetrahedron be OABC so its volume $\frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$

So vertices of new tetrahedron are.

$\frac{\vec{a}+\vec{b}+\vec{c}}{3}, \frac{\vec{b}+\vec{c}}{3}, \frac{\vec{a}+\vec{b}}{3}, \frac{\vec{c}+\vec{a}}{3}$



So volume of new tetrahedron is $\frac{1}{6}\left[\frac{\vec{a}}{3}\ \frac{\vec{b}}{3}\ \frac{\vec{c}}{3}\right]$

$\therefore \frac{\text{volume of given tetrahedron}}{\text{volume of new tetrahedron}} = 27$

22.30 **(2)**

Since A,B,C are non-collinear points, and plane. ABC is not passing through (0, 0, 0) hence $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ are non-coplanar vectors hence $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$ are also non-coplanar vectors.

23.31 **(1)**

S_1 : Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

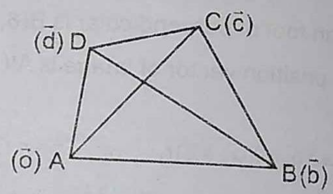
$(\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (a.\hat{k})\hat{k} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$= \vec{a}$

23.32 **(3)**

$\overrightarrow{AB} = \vec{b}$ $\overrightarrow{CD} = \vec{d} - \vec{c}$ $\overrightarrow{BC} = \vec{c} - \vec{b}$

$\overrightarrow{AD} = \vec{b}.$ $\overrightarrow{CA} = -\vec{c}$ $\overrightarrow{BD} = \vec{d} - \vec{b}$



$\therefore \quad |\overrightarrow{AB}\times\overrightarrow{CD} + \overrightarrow{BC}\times\overrightarrow{AD} + \overrightarrow{CA}\times\overrightarrow{BD}|$

$= |\vec{b}\times(\vec{d}-\vec{c}) + (\vec{c}-b)\times\vec{d} - \vec{c}\times(\vec{d}-\vec{b})|$

$= |\vec{b}\times\vec{d} - \vec{b}\times\vec{c} + \vec{c}\times\vec{d} - \vec{b}\times d - \vec{c}\times\vec{d} + \vec{c}\times\vec{b}|$

$= |-\vec{b}\times\vec{c} + \vec{c}\times b| = |-2\vec{b}\times\vec{c}| = 2|\vec{b}\times\vec{c}| = 4$ (area of Δ ABC)

23.33 **(1)**

$S_1 : \begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = 0 \quad \Rightarrow \quad (ab+bc+ca)^3 = 0$

$\Rightarrow \quad ab + bc + ca = 0$

S_2 : True

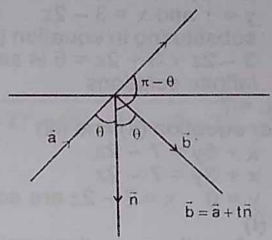
24. 3-D (Three Dimensional Geometry)

24.1 (2)

$\vec{a} = \dfrac{\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{11}}$

$\vec{n} = \dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

$\vec{n}.\vec{a} = \dfrac{1-3+1}{\sqrt{33}} = -\dfrac{1}{\sqrt{33}}$



since $\vec{a}, \vec{b}, \vec{n}$ are coplanar, so

$\vec{b} = \vec{a} + t\vec{n}$

$\vec{b}.\vec{n} = \vec{a}.\vec{n} + t(\vec{n}.\vec{n})$

$t = \vec{b}.\vec{n} - \vec{a}.\vec{n}$ .... (i)

$\vec{b}.\vec{n} = \cos\theta$, $\vec{a}.\vec{n} = \cos(\pi - \theta) \Rightarrow \vec{b}.\vec{n} = -\vec{a}.\vec{n}$

$t = -2(\vec{a}.\vec{n})$

$\therefore \vec{b} = \vec{a} - 2(\vec{n}.\vec{a})\vec{n}$

$\Rightarrow \vec{b} = \dfrac{\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{11}} + \dfrac{2}{\sqrt{33}} \dfrac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}$

$\Rightarrow \vec{b} = \dfrac{1}{3\sqrt{11}}(3\hat{i} - 9\hat{j} + 3\hat{k} + 2\hat{i} + 2\hat{j} + 2\hat{k})$

$\Rightarrow \vec{b} = \dfrac{1}{3\sqrt{11}}(5\hat{i} - 7\hat{j} + 5\hat{k})$

so direction of reflected ray can be (5, – 7 ,5) .

24.2 (3)

Given planes are $\ell x + my = 0$ .... (i)
and $z = 0$ .... (ii)

$\therefore$ equation of any plane passing through the line of intersection of planes (i) and (ii) is -

$\ell x + my + \lambda z = 0$ .... (iii)

DC's of normal to the plane (iii) are

$\dfrac{\ell}{\sqrt{\ell^2 + m^2 + \lambda^2}}, \dfrac{m}{\sqrt{\ell^2 + m^2 + \lambda^2}}, \dfrac{\lambda}{\sqrt{\ell^2 + m^2 + \lambda^2}}$

DC's of normal to the palne (i) are

$\dfrac{\ell}{\sqrt{\ell^2 + m^2}}, \dfrac{m}{\sqrt{\ell^2 + m^2}}, 0$

$\therefore \cos\alpha = \dfrac{\ell.\ell + m.m + 0}{\sqrt{\ell^2 + m^2 + \lambda^2}\sqrt{\ell^2 + m^2}} = \dfrac{\sqrt{\ell^2 + m^2}}{\sqrt{\ell^2 + m^2 + \lambda^2}}$

$\therefore \sec^2\alpha = \dfrac{\ell^2 + m^2 + \lambda^2}{\ell^2 + m^2} = 1 + \dfrac{\lambda^2}{\ell^2 + m^2}$

$\Rightarrow \tan^2\alpha = \dfrac{\lambda^2}{\ell^2 + m^2} \Rightarrow \lambda = \pm\sqrt{(\ell^2 + m^2)}\tan\alpha$

by comparing , we get

$n = \lambda = \pm\sqrt{(\ell^2 + m^2)}\tan\alpha$

24.3 **(4)**

$x + 3y + 2z = 6$ ....(i)
$x + \lambda y + 2z = 7$ ....(ii)
$x + 3y + 2z = \mu$ ....(iii)

(1) If $\lambda = 2$, then $D = 0$, therefore unique solution is not possible

(2) If $\lambda = 4$, $\mu = 6$

$x + 3y = 6 - 2z$
$x + 4y = 7 - 2z$
$\therefore$ $y = 1$ and $x = 3 - 2z$
substituting in equation (iii)
$3 - 2z + 3 + 2z = 6$ is satisfied
$\therefore$ infinite solutions

(3) $\lambda = 5$, $\mu = 7$

consider equation (ii) and (iii)
$x + 5y = 7 - 2z$
$x + 3y = 7 - 2z$
$\therefore$ $y = 0$ $x = 7 - 2z$ are solution
sub. in (i)
$7 - 2z + 2z = 6$ does not satisfy
$\therefore$ no solution

(4) if $\lambda = 3$, $\mu = 5$

then equation (i) and (ii) have no solution
$\therefore$ no solution

24.4 **(1)**

Here $\dfrac{3-(-2)}{1-3} = \dfrac{-6-4}{-2-(-6)} = \dfrac{-8-7}{-2-(-8)}$

$\Rightarrow$ $-\dfrac{5}{2} = -\dfrac{5}{2} = -\dfrac{5}{2}$ collinear

24.5 **(1)**

d.c.s $= \left(\dfrac{6}{\sqrt{36+4+9}}, \dfrac{2}{\sqrt{36+4+9}}, \dfrac{3}{\sqrt{36+4+9}}\right) = \left(\dfrac{6}{7}, \dfrac{2}{7}, \dfrac{3}{7}\right)$

24.6 **(2)**

$\dfrac{a}{1/bc} = \dfrac{b}{1/ca} = \dfrac{c}{1/ab} = abc$ hence lines are ||.

24.7 **(1)**

Line passing through the point $(1, 2, -4)$ is

$\dfrac{x-1}{\ell} = \dfrac{y-2}{m} = \dfrac{z+4}{n}$

according to question $3\ell - 16m + 7n = 0$
and $3\ell + 8m - 5n = 0$

hence required line $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+4}{6}$

24.8 **(1)**

$\dfrac{x-4}{5} = \dfrac{y-1}{2} = \dfrac{z}{1} = r_1$ $\qquad P(5r_1 + 4, 2r_1 + 1, r_1)$

$\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4} = r_2$ $\qquad P(2r_2 + 1, 3r_2 + 2, 4r_2 + 3)$

Now $5r_1 + 4 = 2r_2 + 1$ .....(i)
$2r_1 + 1 = 3r_2 + 2$ .....(ii)
$r_1 = 4r_2 + 3$ .....(iii)

(ii) & (iii) $2(4r_2 + 3) + 1 = 3r_2 + 2$
$5r_2 + 6 + 1 = 2$
$r_2 = -1, r_1 = -1$ it is also satisfied by equation (i) so they intersect
$P(-1, -1, -1)$

24.9 (4)

$L_1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ A (3, 8, 3) $\vec{n}_1 = (3, -1, 1)$

$L_2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ B(−3, −7, 6) $\vec{n}_2$ (−3, 2, 4)

Shortest distance = $\frac{\overrightarrow{AB}.(\vec{n}_1 \times \vec{n}_2)}{|\vec{n}_1 \times \vec{n}_2|} = \begin{vmatrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} \Big/ \sqrt{(-4-2)^2 + (12+3)^2 + (6-3)^2}$

$= 3\sqrt{30}$

24.10 (2)

Equation of plane parallel to y-axis is ax + bz + 1 = 0

(2, 0, 0) 2a + 0 + 1= 0 $\Rightarrow$ $a = -\frac{1}{2}$

(0, 0, 3) 3b + 1 = 0 $\Rightarrow$ $b = -\frac{1}{3}$

$-\frac{x}{2} + \left(\frac{-z}{3}\right) + 1 = 0 = 3x + 2z = 6$

24.11 (3)

Let the equation of the required plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. then the coordinates of A, B, C are (a, 0, 0), (0, b, 0), (0, 0, c) so as the centroid of triangle is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ which is given as (p, q, r)

$\Rightarrow$ a = 3p, b = 3q, c = 3r

hence the plane is $\frac{x}{3p} + \frac{y}{3q} + \frac{z}{3r} = 1$.

24.12 (2)

Let A (1, 0, 0) and B(0, 1, 0)

direction ratio of AB are $\langle -1, 1, 0 \rangle$

Normal is perpendicular to AB and makes an angle $\frac{\pi}{4}$ with the direction $\langle 1, 1, 0 \rangle$

$\therefore$ direction ratios of the normal are $\langle 1, 1, \sqrt{2} \rangle$.

24.13 (4)

By definition of sphere

24.14 (1)

Let equation of the plane be a(x + 1) + b(y − 2) + cz = 0

then 3a − c = 0 and a + 2b − c = 0

Also $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$

$\therefore$ equation of the plane is x + 2y + 3z − 3 = 0

24.15 (2)

x = cy + bz   y = az + cx

z = bx + ay   pass through a line

$$\Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

i.e. $-1(1 - a^2) - c(-c - ab) + b(ac + b) = 0$

$-1 + a^2) + c^2 + abc + abc + b^2 = 0$

$\therefore$ $a^2 + b^2 + c^2 + 2abc = 1$

24.16 (1)

The centre of sphere be P, OP = i – 2j – k

Let centre of circle is Q hence PQ will be perpendicular to the given plane hence

$\overrightarrow{PQ} = \lambda(2i + 2j - k)$

$\overrightarrow{OQ} - \overrightarrow{OP} = \lambda(2i + 2j - k)$

$\overrightarrow{OQ} = \overrightarrow{OP} + \lambda(2i + 2j - k)$

$= (i - 2j - k) + \lambda(2i + 2j - k)$

and point Q lie on the plane.

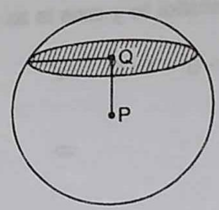
r. (2i + j – k) = 8

$((i - 2j - k) + \lambda(2i + 2j - k)).(2i + 2j - k) = 8$

$\Rightarrow$ $2 - 4 + 1 + \lambda(4 + 4 + 1) = 8$

$\Rightarrow$ $\lambda = 1$

$\overrightarrow{OQ} = i - 2j - k + 2i + 2j - k = 3i - 2k$



24.17 (2)

AB d.r.s. = 3 – 2, 5 – 3, –3 + 1

= 1, 2, – 2

CD d.r.s. = 3 – 1, 5 – 2, 7 – 3

= 2, 3, 4

AB is $\perp$ to CD so projection = 0

24.18 (1)

$\because$ Equation of required plane be $x - y + 2z - 3 + \lambda(4x + 3y - z) = 0$

if we put $\lambda = -3$, we get

$11x + 10y - 5z = 0$ **Ans.**

24.19 (2)

Any point in yz-plane is $(0, \beta, \gamma)$

$\therefore$ $\beta + \gamma = 3$

Its distance from xz-plane = 2. Its distance from xy-plane

$\Rightarrow$ $|\beta| = 2|\gamma|$ $\Rightarrow$ $\beta = \pm 2\gamma$

If $\beta = 2\gamma$, then $\beta = 2$ and $\gamma = 1$

If $\beta = -2\gamma$, then $\beta = 6$ and $\gamma = -3$

(0, 2, 1), (0, 6, –3).

24.20 (4)

Direction ratio of two diganoals are 1, 1, 1 and –1, 1, 1

Let α be the angle between lines, then

$$\cos\alpha = \frac{-1+1+1}{\sqrt{3}.\sqrt{3}} = \frac{1}{3}$$

$$\therefore \quad \alpha = \cos^{-1}\frac{1}{3}$$

24.21 (1)
Let D be the circumcenter of triangle ABC
Direction ratios of BD are <–1, 2, 2>
Direction ratios of AD are <a + 2, –1, 2>
$\because$ AD $\perp$ BD
$\therefore$ $-1\,(a + 2) - 2 + 4 = 0$
$\therefore$ $a = 0$.

24.22 (3)
Let the plane be $ax + by + cz + d + \lambda\,(a'x + b'y + c'z + d') = 0$
since it is parallel to x-axis

$\therefore$ $(a + \lambda a')\,.\,1 = 0$ i.e. $\lambda = -\dfrac{a}{a'}$

$\therefore$ equation of the plane is $ax + by + cz + d - \dfrac{a}{a'}\,(a'x + b'y + c'z + d') = 0$

i.e. $(ab' - a'b)\,y + (ac' - a'c)\,z + (ad' - a'd) = 0$

24.23 (3)
Since $a_1 a_2 + b_1 b_2 + c_1 c_2 = +12 + 72 + 6 > 0$

$\therefore$ the required equation is $\dfrac{3x - 6y + 2z + 5}{\sqrt{9 + 36 + 4}} = \dfrac{-4x + 12y - 3z + 3}{\sqrt{16 + 144 + 9}}$

i.e. $39x - 78y + 26z + 65 = -28x + 84y - 21z + 21$
i.e. $67x - 162y + 47z + 44 = 0$

24.24 (2)
Since PQ is perpendicular to the plane and direction ratio of PQ are $\alpha - \alpha', \beta - \beta', \gamma - \gamma'$
hence equation of plane can be written as

$$(\alpha - \alpha')\,x + (\beta - \beta')\,y + (\gamma - \gamma')\,z = 0$$

also plane passes through $\left(\dfrac{\alpha + \alpha'}{2}, \dfrac{\beta + \beta'}{2}, \dfrac{\gamma + \gamma'}{2}\right)$

hence $(\alpha - \alpha')\left(\dfrac{\alpha + \alpha'}{2}\right) + (\beta - \beta')\left(\dfrac{\beta + \beta'}{2}\right) + (\gamma - \gamma')\left(\dfrac{\gamma + \gamma'}{2}\right) = 0$

$\Rightarrow$ $\alpha^2 + \beta^2 + \gamma^2 = (\alpha')^2 + (\beta')^2 + (\gamma')^2$

24.25 (3)
d.r.s. = 1, 1, 1

Equation of line $\dfrac{(x + 3)}{1} = \dfrac{y - 2}{1} = \dfrac{z + 4}{1}$

or $\dfrac{(x + 3)}{-2} = \dfrac{y - 2}{-2} = \dfrac{z + 4}{-2}$

24.26 (2)
For A(1, 2, 3), $2x - y - 3z - 5 = 2 - 1 - 3 - 5 < 0$
For B(2, 1, –3), $2x - y - 3z - 5 = 4 - 1 + 9 - 5 > 0$
For C(1, –2, –2), $2x - y - 3z - 5 = 2 + 2 + 6 - 5 > 0$
For D(–3, 1, 2), $2x - y - 3z - 5 = -6 - 1 - 6 - 5 < 0$
$\therefore$ AD are on one side of the plane and B, C are on the other side
$\therefore$ the line segments AB, AC, BD, CD intersect the plane.

24.27 **(2)**

Equation of the plane is

$$\begin{vmatrix} x & y-7 & z+7 \\ -3 & 2 & 1 \\ -(-1) & 7-3 & -7-(-2) \end{vmatrix} = 0$$

i.e. $\begin{vmatrix} x & y-7 & z+7 \\ -3 & 2 & 1 \\ 1 & 4 & -5 \end{vmatrix} = 0$

i.e. $x(-10-4) - (y-7)(15-1) + (z+7)(-12-2) = 0$

i.e. $x + y - 7 + z + 7 = 0$

i.e. $x + y + z = 0$

24.28 **(1)**

Direction ratios of the lines are $<1, 2, 3>$ and $<-2, -4, -6>$

Thus $-\frac{2}{1} = -\frac{4}{2} = -\frac{6}{3}$

$\therefore$ the lines are parallel. Also (1, 2, 3) lies on both the lines

$\therefore$ the lines are coincident

24.29 **(2)**

Point of intersection of the line and the plane is $(-2, -2, -5)$ ....(i)

image of $(1, 2, -2)$ in the plane $2x + 5y - 4z - 6 = 0$ is

$\left(-\frac{11}{45}, -\frac{10}{9}, \frac{22}{45}\right)$

$\therefore$ equation of the image line is $\frac{x+2}{79} = \frac{y+2}{40} = \frac{z+5}{247}$

24.30 **(3)**

Two lines are coplanar if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

24.31 **(4)**

Using distance formula, we shall find that $|AB| = |BC| = |CD| = |DA| = 2$.

So, the statement -2 is correct.

We note that the points A,B,C lie in the plane $z = 0$ as each of these points has z-coordinate = 0.

However, the point D does not lies in this plane.

$\therefore$ The four points A,B,C, D are non-coplanar, therefore, they cannot make a square.

24.32 **(1)**

We note that the point (1, 2, 3) lies on both the lines. So, the two linesintersect at the point (1, 2, 3). Also their d.n.s. are not proportional. So the two lines are not coincident. Thus $\ell_1 \cap \ell_2$= a singleton set, consequently, the S.D. between ℓ_1 and ℓ_2 is 0.

24.33 **(4)**

Here, the centre of the sphere S is (0, 0, 0) and its radius is 5. We write p in the normal form by dividing its equation throught by

$\sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

So, p is $\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}y + \frac{1}{\sqrt{3}}z = 4$, which is of the form $\ell x + my + nz = p$, where $p = 4$

25. TRIGONOMETRIC IDENTITIES & EQUATION

25.1 (2)

We have

$$\frac{\sin x + 1}{\cos x} = 2\cos x$$

$\therefore\ 2\cos^2 x = \sin x + 1$

$\Rightarrow \sin x = -1$ or $\frac{1}{2}$

$$\left\{\begin{matrix}\text{but } \sin x \neq -1 \\ \because \cos x \neq 0\end{matrix}\right\}$$

$\therefore\ \sin x = \frac{1}{2}$

$\therefore$ In $[0, 2\pi]$

$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

two solutions.

25.2 (2)

In ΔABC

$b+c-a > 0$, $c+a-b > 0$, $a + b - c > 0$, so

$$\frac{(b+c-a)+(c+a-b)+(a+b-c)}{3} \geq \{(b+c-a)(c+a-b)(a+b-c)\}^{1/3}$$

$$\Rightarrow \frac{P}{3} \geq \{(b+c-a)(c+a-b)(a+b-c)\}^{1/3}$$

$\Rightarrow P^3 \geq 27\,(b+c-a)\,(c+a-b)\,(a+b-c)$ .... (i)

Also $\frac{(a+b+c)+(b+c-a)+(c+a-b)+(a+b-c)}{4} \geq \{(a+b+c)\,(b+c-a)\,(c+a-b)\,(a+b-c)\}^{1/4}$

$$\Rightarrow \frac{2P}{4} \geq (16A)^{1/4}$$

$\Rightarrow P \geq 4A^{1/4}$

$\Rightarrow P^4 \geq 256A$

For a given perimeter equilateral Δ has the greatest area. So area of any triangle

$$A \leq \frac{\sqrt{3}}{4}\left(\frac{P}{3}\right)^2$$

$A \leq \frac{\sqrt{3}}{36}P^2$ .... (ii)

Now $P^2 = (a+b+c)^2$

$= a^2+b^2+c^2 + 2(ab+bc+ca)$

$=3(a^2+b^2+c^2) - 2(a^2+b^2+c^2-ab-bc-ca)$

$P^2 \leq 3(a^2+b^2+c^2)$ .... (iii)

$$\therefore\ A \leq \frac{\sqrt{3}}{36}\,3\,(a^2+b^2+c^2)$$

$$a^2 + b^2 + c^2 \geq 4\sqrt{3}\,A$$

25.3 (2)

$$\frac{1-2.(\cos 60^\circ - \cos 80^\circ)}{2\sin 10^\circ}$$

$$\frac{1-2\left(\frac{1}{2}\right)+2\cos 80^\circ}{2\sin 10^\circ} = 1$$

25.4 (2)

$$\tan A = \frac{1-\cos B}{\sin B} = \frac{2\sin^2\frac{B}{2}}{2\sin\frac{B}{2}\cos\frac{B}{2}} = \tan\frac{B}{2}$$

$$\therefore \quad \tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\tan\frac{B}{2}}{1-\tan^2\frac{B}{2}} = \tan B$$

25.5 (3)

$$\sqrt{\frac{1-\sin x}{1+\sin x}} + \sqrt{\frac{1+\sin x}{1-\sin x}}$$

$$\sqrt{\frac{1-\sin x}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}} + \sqrt{\frac{1+\sin x}{1-\sin x}\times\frac{1+\sin x}{1+\sin x}}$$

$$= \left|\frac{1-\sin x}{\cos x}\right| + \left|\frac{1+\sin x}{\cos x}\right|$$

$$= -\left(\frac{1-\sin x}{\cos x}\right) - \frac{1+\sin x}{\cos x} \qquad \text{(as } x \in \text{ II quadrant)}$$

$$= -\frac{2}{\cos x}$$

25.6 (3)

$x = \sin^8\theta + \cos^{14}\theta$

$= (\sin^2\theta)^4 + (\cos^2\theta)^7 \le \sin^2\theta + \cos^2\theta$ (as $\sin^2\theta \in [0, 1]$) and $\cos^2\theta \in [0, 1]$

$\Rightarrow \quad (\sin^2\theta)^4 + (\cos^2\theta)^7 \le 1$

$\Rightarrow \quad x \le 1$

But also $x > 0$ ($x \ne 0$ because for $x = 0$, $\sin^2\theta = 0$ & $\cos^2\theta = 0$, which is not possible simultaneously)

$\therefore \quad 0 < x \le 1$

25.7 (4)

Let $\quad 1 - \cos\theta = t$

$\Rightarrow \quad 1 + 2t + 3t^2 + 4t^3 + \ldots\ldots$

$$= (1-t)^{-2} = [1-(1-\cos\theta)]^{-2} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

$$= 1 + \tan^2\theta = 1 + \frac{3}{2} = \frac{5}{2}$$

25.8 (2)

Solving given det, $(c^2 - ab) - a(c - a) + b(b - c) = 0$

$\Rightarrow$ $a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow$ $(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

$\Rightarrow$ $a - b = 0,\ b - c = 0,\ c - a = 0$

$\Rightarrow$ $a = b = c$ $\Rightarrow$ Δ is equilateral

$\Rightarrow$ $\angle A = \angle B = \angle C = 60^\circ$

$\therefore$ $\sin^2 A + \sin^2 B + \sin^2 C = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$

25.9 (1)

$\tan 70^\circ = \tan(20^\circ + 50^\circ) = \frac{\tan 20^\circ + \tan 50^\circ}{1 - \tan 20^\circ \tan 50^\circ}$

$\Rightarrow$ $\tan 70^\circ - \tan 20^\circ \tan 50^\circ \tan 70^\circ = \tan 20^\circ + \tan 50^\circ$

$\Rightarrow$ $\tan 70^\circ - \tan 20^\circ \tan 50^\circ \cot 20^\circ = \tan 20^\circ + \tan 50^\circ$

$\Rightarrow$ $\tan 70^\circ = \tan 20^\circ + 2\tan 50^\circ$

25.10 (3)

$$2 + (\tan^4\phi + \cot^4\phi) + \left(\frac{1 + \cos 2\theta}{2}\right) - 3.(\sin 2\theta) + 3\left(\frac{1 - \cos 2\theta}{2}\right)$$

$$2 + \left(\tan^4\phi + \frac{1}{\tan^4\phi}\right) + 2 - (\cos 2\theta + 3\sin 2\theta)$$

$\therefore$ least value $= 4 + 2 - (\sqrt{1^2 + (3)^2}) = 6 - \sqrt{10}$

25.11 (3)

$$\cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7}$$

$$= \frac{\cos\left[\dfrac{\frac{\pi}{7} + \frac{6\pi}{7}}{2}\right]\sin\left[6.\frac{\pi}{7} \times \frac{1}{2}\right]}{\sin\left(\frac{\pi}{7} \times \frac{1}{2}\right)}$$

$$= \frac{\cos\frac{\pi}{2}.\sin\frac{3\pi}{7}}{\sin\frac{\pi}{14}} = 0$$

25.12 (1)

$$\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + \ldots.. + \sin(2n - 1)\frac{\pi}{n}$$

$$= \frac{\sin\left(\frac{\pi}{2n} + (2n - 1)\frac{\pi}{2n}\right)\sin\left(n.\frac{2\pi}{n} \times \frac{1}{2}\right)}{\sin\left(\frac{2\pi}{n} \times \frac{1}{2}\right)}$$

$$= \frac{\sin\pi\sin(\pi)}{\sin\frac{2\pi}{2n}} = 0$$

25.13 (2)

$$-\sqrt{2^2+(-3)^2} \le k \le \sqrt{2^2+(-3)^2} \Rightarrow |k| \le \sqrt{13}$$

25.14 (1)

Given equation is true only when

$\cos(\pi\sqrt{x-4}) = -1$ and $\cos(\pi\sqrt{x}) = -1$

or $\cos(\pi\sqrt{x-4}) = 1$ and $\cos(\pi\sqrt{x}) = 1$

$\Rightarrow$ either $\pi\sqrt{x-4}$ and $\pi\sqrt{x}$ both are even integer

or both are odd integer

$\Rightarrow$ x = 4 is the only solution

$\therefore$ number of solution = 1

25.15 (3)

(sin 5x + sinx) + sin 3x = 0

$\Rightarrow$ 2sin 3x cos 2x + sin 3x = 0

$\Rightarrow$ sin 3x (2 cos 2x + 1) = 0

$\Rightarrow$ sin 3x = 0 or $\cos 2x = -\frac{1}{2}$

$\Rightarrow$ $3x = 0, \pi, 2\pi, 3\pi, \ldots\ldots$ $\Rightarrow$ $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \ldots..$

or $2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \ldots\ldots$ $\Rightarrow$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$

but given $x \in \left(0, \frac{\pi}{2}\right]$ $\therefore$ $x = \frac{\pi}{3}$

25.16 (2)

cos 2x ≤ – sin x

$\Rightarrow$ $1 - 2\sin^2 x \le -\sin x$ $\Rightarrow$ $(2\sin x + 1)(\sin x - 1) \ge 0$

$\Rightarrow$ $\sin x \le -\frac{1}{2}$ or $\sin x \ge 1$

$\Rightarrow$ $-\frac{\pi}{2} \le x \le -\frac{\pi}{6}$ or $x = (4n+1)\frac{\pi}{2}$ (as sinx = 1 is valid only)

$\Rightarrow$ In general $x \in \left[2n\pi - \frac{\pi}{2}, 2n\pi - \frac{\pi}{6}\right] \cup \left[(4n+1)\frac{\pi}{2}\right]$

25.17 (1)

sin x + sin 5x = sin 2x + sin 4x

$\Rightarrow$ 2 sin 3x cos 2x = 2 sin 3x cos x

$\Rightarrow$ 2 sin 3x(cos 2x – cos x) = 0

$\Rightarrow \sin 3x = 0 \Rightarrow 3x = n\pi \Rightarrow x = \frac{n\pi}{3}$

or cos 2x – cos x = 0 $\Rightarrow$ cos 2x = cos x

$\Rightarrow 2x = 2n\pi \pm x \Rightarrow x = 2n\pi, \frac{2n\pi}{3}$

But solutions obtained by $x = 2n\pi$ or $x = \frac{2n\pi}{3}$ are all involved in $x = \frac{n\pi}{3}$

5.18 (4)
$\sin 3\theta = 2\sin\theta\,(2\sin 2\theta \sin 4\theta)$
$\Rightarrow \sin 3\theta = 2\sin\theta\,(\cos 2\theta - \cos 6\theta)$
$\Rightarrow \sin 3\theta = 2\sin\theta\cos 2\theta - 2\sin\theta\cos 6\theta$
$\Rightarrow \sin 3\theta = \sin 3\theta - \sin\theta - 2\sin\theta\cos 6\theta$
$\Rightarrow \sin\theta\,(1 + 2\cos 6\theta) = 0$
$\Rightarrow \sin\theta = 0\,,\ 1 + 2\cos 6\theta = 0$

$$\Rightarrow \theta = n\pi,\ 6\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = \frac{n\pi}{3} \pm \frac{\pi}{9}$$

$$\Rightarrow \theta = 0, \pi\ , \theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{5\pi}{9}, \frac{8\pi}{9} \quad (\text{as } 0 \le \theta \le \pi)$$

$\therefore$ total number of solutions = 8

25.19 (4)

$$P = \cos\frac{\pi}{10}\cos\frac{2\pi}{10}\cos\frac{4\pi}{10}\cos\frac{8\pi}{10}\cos\frac{16\pi}{10}$$

$$= \frac{\sin 2^5.\frac{\pi}{10}}{2^5\sin\frac{\pi}{10}} = \frac{\sin 32\frac{\pi}{10}}{32\sin\frac{\pi}{10}} = \frac{\sin\left(3\pi + \frac{2\pi}{10}\right)}{32\sin\left(\frac{\pi}{10}\right)} = -\frac{\sin\frac{2\pi}{10}}{32\sin\frac{\pi}{10}}$$

$$= -\frac{\sin\frac{\pi}{5}}{32\sin\frac{\pi}{10}} = -\frac{2\sin\frac{\pi}{10}\cos\frac{\pi}{10}}{32\sin\frac{\pi}{10}} = -\frac{1}{16}\left(\cos\frac{\pi}{10}\right)$$

$$= -\frac{1}{16}\cos 18^\circ = -\frac{1}{16}\left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right) = -\frac{1}{64}\sqrt{(10+2\sqrt{5})}$$

25.20 (4) $P = \sqrt{2+\sqrt{2+\sqrt{2+2\cos 4\theta}}}$

$$= \sqrt{2+\sqrt{2+\sqrt{2(2\cos^2 2\theta)}}} = \sqrt{2+\sqrt{2+|2\cos 2\theta|}}$$

given $0 < \theta < \frac{\pi}{2}$ $\Rightarrow$ $0 < 2\theta < \pi$

case- 1 if $0 < 2\theta < \frac{\pi}{2}$

then $P = \sqrt{2+\sqrt{2+2\cos 2\theta}}$

$= \sqrt{2+\sqrt{2.(2\cos^2\theta)}}$

$= \sqrt{2+|2\cos\theta|}$

$\because$ $0 < \theta < \frac{\pi}{2} \Rightarrow |\cos\theta| = \cos\theta$

$\therefore$ $P = \sqrt{2+2\cos\theta}$

$= \sqrt{2\left(2\cos^2\frac{\theta}{2}\right)}$

$= |2\cos\frac{\theta}{2}|$

$= 2\cos\frac{\theta}{2}$ as $0 < \frac{\theta}{2} < \frac{\pi}{4}$

case - 2 if $\frac{\pi}{2} < 2\theta < \pi$

then, $P = \sqrt{2+\sqrt{2-(2\cos 2\theta)}}$

$= \sqrt{2+\sqrt{2(1-\cos 2\theta)}}$

$= \sqrt{2+\sqrt{2.2\sin^2\theta}}$

$= \sqrt{2+|2\sin\theta|}$

$= \sqrt{2+2\sin\theta}$ as $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

$= \sqrt{2(1+\sin\theta)}$

$= \sqrt{2\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2}$

$= \sqrt{2}\left|\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right|$

$= \sqrt{2}\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)$ as $\frac{\pi}{8} < \frac{\theta}{2} < \frac{\pi}{4}$

25.21 (2)

$$\cos\frac{4\pi}{5}\cos\frac{6\pi}{5}\cos\frac{8\pi}{5} = \cos\left(\frac{4\pi}{5}\right)\left[-\cos\frac{\pi}{5}\right]\cos\left(\frac{8\pi}{5}\right)$$

$$= \frac{-\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}}{\cos\frac{2\pi}{5}} = -\frac{\sin\left(2^4\frac{\pi}{5}\right)}{2^4\sin\left(\frac{\pi}{5}\right)\cos\frac{2\pi}{5}}$$

$$= -\frac{\sin\left(3\pi+\frac{\pi}{5}\right)}{16\sin\left(\frac{\pi}{5}\right)\cos 72^\circ} = \frac{\sin\left(\frac{\pi}{5}\right)}{16\sin\left(\frac{\pi}{5}\right)\sin 18^\circ} = \frac{1}{16\left(\frac{\sqrt{5}-1}{4}\right)} = \frac{1}{4(\sqrt{5}-1)}$$

Also $\dfrac{\sin\left(\frac{\pi}{5}\right)}{16\sin\left(\frac{\pi}{5}\right)\cos 72^\circ} = \dfrac{1}{16}\sec 72^\circ$ $\quad\therefore\quad$ option (2) & (3) are correct.

25.22 (4)

$$\frac{\cos[-(90^\circ-\theta)]\sec\theta[\tan(180^\circ-\theta)]}{\sec(360^\circ-\theta)[\sin\{-(180^\circ-\theta)\}]\tan(360^\circ-\theta)} = \frac{\sin\theta.\sec\theta(-\tan\theta)}{\sec\theta(-\sin\theta)(-\tan\theta)} = -1$$

Also $[-\tan 45^\circ \tan 15^\circ \tan 255^\circ)]$

$= -\ [\tan 45^\circ \tan 15^\circ \tan(180^\circ + 75^\circ)]$

$= -\tan 15^\circ \tan(60^\circ - 15^\circ)\tan(60^\circ + 15^\circ)$

$= -\tan(3\times 15^\circ)$

$= -1$

25.23 (4)

$1 + \sin x + \sin^2 x + \ldots\ldots$ upto $\infty = 4 + 2\sqrt{3}$

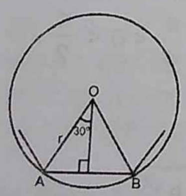
$\Rightarrow \dfrac{1}{1-\sin x} = (4+2\sqrt{3}) \Rightarrow 1-\sin x = \dfrac{1}{4+2\sqrt{3}}\times\dfrac{4-2\sqrt{3}}{4-2\sqrt{3}}$

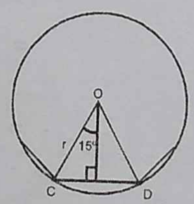
$\Rightarrow 1-\sin x = \dfrac{4-2\sqrt{3}}{4} \Rightarrow \sin x = \dfrac{2\sqrt{3}}{4} = \dfrac{\sqrt{3}}{2}$

$\therefore x = \dfrac{\pi}{3}$ or $\pi - \dfrac{\pi}{3} \Rightarrow x = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

25.24 (4)

For Hexogon $\angle AOB = \dfrac{360^\circ}{6} = 60^\circ$ $\qquad$ for Dodecagon : $\angle COD = \dfrac{360^\circ}{12} = 30^\circ$





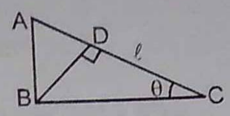
$AB = 2r\sin 30^\circ$ $\qquad$ $CD = 2r\sin 15^\circ$

$$\frac{CD}{AB} = \frac{\sin 15^\circ}{\sin 30^\circ}$$

$$AB = \frac{CD\sin 30}{\sin 15} = \frac{(\sqrt{3}-1)\times\frac{1}{2}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \sqrt{2}$$

25.25 **(2)**
BC = $\ell \cos\theta$
BD = BC $\sin\theta$ = $\ell \sin\theta \cos\theta$

$\ell \sin\theta \cos\theta = \frac{\ell}{2\sqrt{2}}$



$\sin 2\theta = \frac{1}{\sqrt{2}}$

$2\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$\theta = \frac{\pi}{8}, \frac{3\pi}{8}$

25.26 **(4)**

$$\frac{1}{\tan x} + \frac{1-\sqrt{3}\tan x}{\tan x+\sqrt{3}} + \frac{1+\sqrt{3}\tan x}{\tan x-\sqrt{3}}$$

$$= \frac{\tan^2 x - 3 + \tan x\left(1-\sqrt{3}\tan x\right)\left(\tan x-\sqrt{3}\right) + \tan x\left(1+\sqrt{3}\tan x\right)\left(\tan x+\sqrt{3}\right)}{\tan x\left(\tan^2 x-3\right)}$$

$$= \frac{\tan^2 x - 3 + \tan x\left(-\sqrt{3}\tan^2 x + 4\tan x - \sqrt{3} + \sqrt{3}\tan^2 x + 4\tan x + \sqrt{3}\right)}{\tan x\left(\tan^2 x-3\right)} = \frac{3(1-3\tan^2 x)}{\tan x(3-\tan^2 x)}$$

25.27 **(2)**

$\frac{\cos 3\theta}{\cos\theta} = 4\cos^2\theta - 3 \quad = \quad 2(1+\cos 2\theta) - 3$

$= \quad 2\cos 2\theta - 1$

$\cos\alpha + \cos\beta = a$ (i)
$\sin\alpha + \sin\beta = b$ (ii)

squaring and adding
$2 + 2\cos(\alpha-\beta) = a^2 + b^2$
$2\cos 2\theta = a^2 + b^2 - 2$
$\Rightarrow \quad 2\cos 2\theta - 1 = a^2 + b^2 - 3$

25.28 **(2)**

$$\frac{1}{\sin 20} - \frac{1}{\sqrt{3}\cos 20^\circ}$$

$$= \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sqrt{3}\sin 20^\circ \cos 20^\circ}$$

$$= \frac{2.(\cos 30^\circ \cos 20^\circ - \sin 30^\circ \sin 20^\circ)}{\sqrt{3}\sin 20^\circ \cos 20^\circ}$$

$$= \frac{2.\cos 50^\circ}{\sqrt{3}\sin 20^\circ \cos 20^\circ}$$

$$= \frac{2\sin 40^\circ}{\sqrt{3}\sin 20^\circ \cos 20^\circ}$$

$$= \frac{4}{\sqrt{3}} \frac{\sin 20^\circ \cos 20^\circ}{\sin 20^\circ \cos 20^\circ}$$

$$= \frac{4\sqrt{3}}{3}$$

25.29 (1)

$\cos A \cos B \cos C = \lambda\,(4(\cos^3 A + \cos^3 B + \cos^3 C) - 3(\cos A + \cos B + \cos C))$

$= 4\lambda\,(\cos^3 A + \cos^3 B + \cos^3 C)$

$= 4\lambda \,.\, 3\cos A \cos B \cos C$

$\Rightarrow 12\lambda = 1$

$\lambda = \dfrac{1}{12}$

25.30 (3)

$\cos\dfrac{3\pi}{7} \,.\, \cos\dfrac{2\pi}{7} \,.\, \cos\dfrac{\pi}{7}$

$= -\cos\dfrac{\pi}{7} \,.\, \cos\dfrac{2\pi}{7} \,.\, \cos\dfrac{4\pi}{7}$

$= \dfrac{-\sin\dfrac{8\pi}{7}}{2^3 \,.\, \sin .\dfrac{\pi}{7}} = \dfrac{1}{8}$

25.31 (2)

$$y = \frac{1}{\frac{3}{2}\sin 2\theta + \frac{5}{2}(1+\cos 2\theta) + \left(\frac{1-\cos 2\theta}{2}\right)}$$

$$= \frac{1}{\frac{3}{2}\sin 2\theta + \frac{4\cos 2\theta}{2} + 3}$$

$$= \frac{1}{3 + \frac{3\sin 2\theta + 4\cos 2\theta}{2}}$$

$$y_{max} = \frac{1}{3 + \left.\frac{3\sin 2\theta + 4\cos 2\theta}{2}\right|_{min.}}$$

$$y_{max} = \frac{1}{3 - \frac{5}{2}} = 2$$

25.32 (3)

Using $\tan\theta = \cot\theta - 2\cot 2\theta$

we get $E = (\cot\theta - 2\cot 2\theta) + 2(\cot 2\theta - 2\cot 4\theta) + 4(\cot 4\theta - 2\cot 8\theta) + \ldots\ldots$

$\ldots + 2^{14}(\cot 2^{14}\theta - 2\cot 2^{15}\theta) + 2^{15}\cot 2^{15}\theta$

$= \cot\theta$

25.33 (1)

$\cos 2A + \cos 2B + \cos 2C$

$= 2\cos(A+B)\cos(A-B) + 2\cos^2 C - 1$

$= -2\cos C\cos(A-B) + 2\cos^2 C - 1 \quad (\because \text{ given } A + B - C = 3\pi)$

$= -2\cos C\,[\cos(A-B) - \cos C] - 1$

$= -2\cos C\,[\cos(A-B) + \cos(A+B)] - 1$

$= -2\cos C\,[2\cos A \cos B] - 1$

$= -4\cos A \cos B \cos C - 1$

25.34 (4)

Statement-2 is true

for statement (1) $2x = 2n\pi + \frac{\pi}{2}$ (tan 2x is not defined)

25.35 (1)

Statement - 1 $\sin 85° \sin 35° \sin 25°$

$= \cos 5° \cos 55° \cos 65°$

$= \cos 5° \cos (60° - 5°) \cos (60° + 5°) = \frac{1}{4} \cos (3 \times 5°) = \frac{1}{4} \times \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore$ statement - 1 is true.

Statement -2 $\cos \theta \cos (60° - \theta)\cos (60° + \theta) = \frac{1}{4} \cos 3\theta$

$= \frac{1}{4}(4\cos^3 \theta - 3 \cos\theta)$

$= \cos^3 \theta - \frac{3}{4} \cos\theta$

$\therefore$ statement-2 is also true and clearly explain the statement-1

25.36 (1)

Statement -1 $8^c = \left(8 \times \frac{180}{\pi}\right)^{\circ} = \left(\frac{8 \times 180 \times 7}{22}\right)^{\circ} = (458.18)°$

Now $0.18° = 0.18 \times 60$ minutes = 10.8 minutes

and 0.8 minutes = 0.8×60 seconds = 48 seconds

$\therefore$ $8^c = 458° 10'48''$

$\therefore$ statement - 1 is true

obiously statement-2 is true and explains statement -1

25.37 (1)

Statment-1 : Let $x - 1 = u$ then $\cos u = \frac{|u|}{10}$

clearly graph of $y = \cos u$ & $y = \frac{|u|}{10}$ cut each other at six real points hence total no. of solutions is 6.

Hence statment-1 is true.

Statement-2 is obviously true and also explains statement-1.

25.38 (1)Obvious

25.39 (3)

$\frac{\cos A \sin A + \cos B \sin B + \cos C \cdot \sin C}{\sin A \sin B \sin C}$

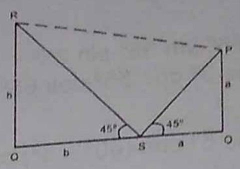
$= \frac{1}{2}\left(\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A \sin B \sin C}\right)$

$= \frac{1}{2} \cdot \left(\frac{4 \sin A \sin B \sin C}{\sin A \sin B \sin C}\right)$

In a ΔABC $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

26. Solution of Triangle and Height distance

26.1 (3)

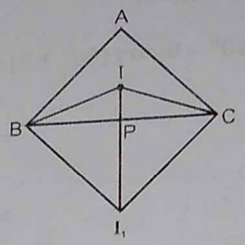


OS = a, QS = b

PS = $\sqrt{2}a$ RS = $\sqrt{2}b$

$PR^2 = PS^2 + RS^2 = 2(a^2 + b^2)$

26.2 (1)



$ICI_1 = \frac{\pi}{2}$

$IBI_1 = \frac{\pi}{2}$,

$\therefore$ $BICI_1$ is cyclic Quadrilateral

$BP . PC = IP . I_1P$

26.3 (1)

$$\cos A + \cos B + \cos C = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} + \cos C$$

$$= 2\sin\frac{C}{2}\cos\frac{A-B}{2} + 1 - 2\sin^2\frac{C}{2}$$

$$= 1 + 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right)$$

$$= 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = 1 + \frac{r}{R}$$

26.4 (4)

$\because \sin B = \frac{b\sin A}{a}$ and $A < \frac{\pi}{2}$

when $b\sin A = a$, $\sin B = 1$, $B = \frac{\pi}{2}$ (possible)

when $\sin B < 1$

$\frac{b\sin A}{a} < 1$

$b\sin A < a$

If $b < a$, then one triangle possible

If $b > a$, then two triangle possible

26.5 (3)

Length of AD = $\sqrt{\frac{2b^2+2c^2-a^2}{4}}$

BE = $\sqrt{\frac{2c^2+2a^2-b^2}{4}}$ $\Rightarrow$ CF = $\sqrt{\frac{2a^2+2b^2-c^2}{4}}$

Now $AD^2 + BE^2 + CF^2 = \frac{2b^2+2c^2-a^2+2c^2+2a^2-b^2+2a^2+2b^2-c^2}{4}$

$= \frac{3(a^2+b^2+c^2)}{4}$

$\therefore \frac{AD^2+BE^2+CF^2}{BC^2+CA^2+AB^2} = \frac{3}{4}$

26.6 (4)

Since $\frac{\sin A}{\sin B} = \frac{a}{b}$ (In any triangle)

$\Rightarrow$ $\sin B = \frac{10}{7} \cdot \frac{3}{4} = \frac{30}{28}$ which is not possible

So no triangle possible.

26.7 (4)

Angles are 30°, 60°, 90° (Let C, B, A),

Let a = 10, b = 9

then $\angle A = \frac{\pi}{2}, \angle B = \frac{\pi}{3}, \angle C = \frac{\pi}{6}$

Now $\frac{\sin B}{\sin C} = \frac{b}{c} \Rightarrow \frac{\sin\frac{\pi}{3}}{\sin\frac{\pi}{6}} = \frac{9}{c}$

$\Rightarrow \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{9}{c} \Rightarrow c = 3\sqrt{3}$

26.8 (1)

$r_1 > r_2 > r_3 \Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c} \Rightarrow \frac{1}{s-a} > \frac{1}{s-b} > \frac{1}{s-c}$

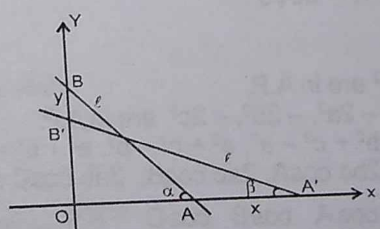
$\Rightarrow s-c > s-b > s-a \Rightarrow -c > -b > -a \Rightarrow a > b > c$

26.9 (1)

Let AB be the original position of the ladder. After sliding, it takes the position A′B′

AA′ = x, BB′ = y, AB = A′B′ = ℓ (say)

Now $\cos\alpha = \frac{OA}{\ell}$ and $\cos\beta = \frac{OA'}{\ell}$



$\therefore \cos\beta - \cos\alpha = \frac{OA'-OA}{\ell} = \frac{x}{\ell}$

Also $\sin\alpha = \frac{OB}{\ell}$ and $\sin\beta = \frac{OB'}{\ell}$

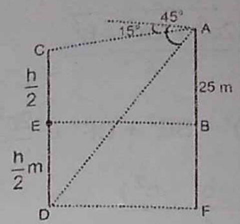
$\Rightarrow \sin\alpha - \sin\beta = \frac{OB-OB'}{\ell} = \frac{y}{\ell}$

Then $\frac{\cos\beta-\cos\alpha}{\sin\alpha-\sin\beta} = \frac{x/\ell}{y/\ell} = \frac{x}{y}$

26.10 **(2)**

Let the tower be AB = 25m and the pole be CD = h m whose middle point be E. So CE = DE = $\frac{h}{2}$ m

$\angle ADF = 45°$, AF = $\left(25+\frac{h}{2}\right)$m



$\therefore \quad \sin 45° = \frac{AF}{AD} = \frac{25+h/2}{AD} \quad \Rightarrow \quad AD = \sqrt{2}\left(25+\frac{h}{2}\right)$m

Now, in the ΔACD

$\angle DAC = 30°$, $\angle ACD = 15° + 90° = 105°$

$\therefore \quad \frac{AD}{\sin 105°} = \frac{CD}{\sin 30°} \quad$ or $\quad \frac{\sqrt{2}\left(25+\frac{h}{2}\right)}{\sin(60°+45°)} = \frac{h}{1/2}$

26.11 **(1)**

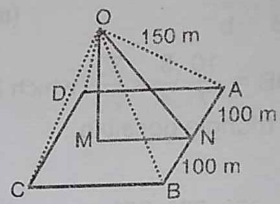
Here OA = OB = OC = OD = 150 m

AB = BC = CD = DA = 200 m

Let OM $\perp$ base.

Then M is the centre of the square.

MN $\perp$ AB.



Clearly AN = NB = 100 ; MN = $\frac{1}{2}$ AD = 100 m.

as ΔOAB is isosceles, ON $\perp$ NA

In ΔOAN, OA = 150 m and NA = 100 m

$ON = \sqrt{150^2 - 100^2} = 50\sqrt{5}$ m.

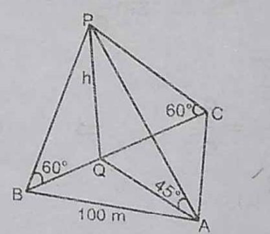
In ΔOMN, $OM = \sqrt{ON^2 - MN^2} = \sqrt{12500 - 10000} = 50$m

26.12 **(2)**

Let PQ = h m

From ΔPQA, QA = h cot 45° = h

From ΔPQB ; BQ = h cot 60° = $\frac{h}{\sqrt{3}}$



from ΔPQC ; CQ = h cot 60° = $\frac{h}{\sqrt{3}}$

$\therefore$ BQ = CQ $\Rightarrow$ Q is the middle point of BC

Also AB = AC = 100 m ;

So ΔABC is isosceles.

$\therefore$ from geometry AQ $\perp$ BC.

From ΔAQB

$BQ^2 + AQ^2 = AB^2 \quad \Rightarrow \quad \frac{h^2}{3} + h^2 = 100^2$

$\Rightarrow \quad h = 50\sqrt{3}$

26.13 **(3)**

a^2, b^2, c^2 are in A.P.

$\Rightarrow$ $-2a^2, -2b^2, -2c^2$ are in A.P.

$\Rightarrow$ $b^2 + c^2 - a^2, a^2 + c^2 - b^2, a^2 + b^2 - c^2$ are in A.P.

$\Rightarrow$ 2bc cosA, 2ac cosB, 2ab cosC are in A.P.

$\Rightarrow$ $\frac{\cos A}{a}, \frac{\cos B}{b}, \frac{\cos C}{c}$ are in A.P.

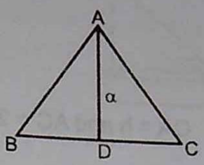
$\Rightarrow$ $\frac{\cos A}{2R\sin A}, \frac{\cos B}{2R\sin B}, \frac{\cos C}{2R\sin C}$ are in A.P.

$\Rightarrow$ cotA, cotB, cotC are in A.P.

$\Rightarrow$ tanA, tanB, tanC are in H.P.

26.14 (2)

Clearly $\frac{1}{2}\alpha.a = \Delta$



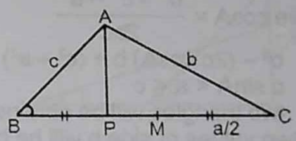
$\therefore \quad \frac{1}{\alpha} = \frac{a}{2\Delta}$

Similarly $\frac{1}{\beta} = \frac{b}{2\Delta}, \ \frac{1}{\gamma} = \frac{c}{2\Delta}$

$\Rightarrow \quad \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{a^2 + b^2 + c^2}{4\Delta^2}$

26.15 (3)

The required distance = MP

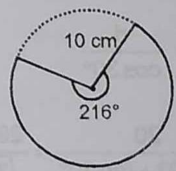


$= \frac{a}{2} - BP = \frac{a}{2} - c \cos B = \frac{a}{2} - c\left(\frac{c^2 + a^2 - b^2}{2ac}\right)$

$= \frac{a^2 - (c^2 + a^2 - b^2)}{2a} = \frac{b^2 - c^2}{2a}$

26.16 (1)

The area of the sector = $\frac{\pi.10^2}{360} \times 216 \text{ cm}^2$



= lateral surface area of cone

The circumference of the base circle of the cone

$= \frac{2\pi \times 10}{360} \times 216 \text{ cm}$

$2\pi r = \frac{2\pi \times 10}{360} \times 216 \quad$ or $\quad r = 6$ cm

The lateral height of the cone = 6 cosec θ cm

$\therefore \quad$ lateral surface area = $\pi r\ell = \pi.6.6 \text{ cosec}\theta. \text{ cm}^2$

$\Rightarrow \quad \pi\, 36 \,.\, \text{cosec}\,\theta = \frac{\pi.10^2.216}{360} \quad \Rightarrow \quad \sin\theta = \frac{3}{5}$

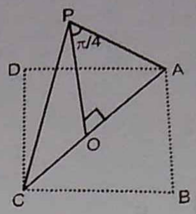
26.17 (4)

If the height of the pole is h metres then $\frac{h}{\text{circum radius}} = \tan 60°$

$\therefore \quad$ circumradius = $\frac{h}{\sqrt{3}}$ metres $\quad \therefore \quad A = \pi\left(\frac{h}{\sqrt{3}}\right)^2$ metre²

The area of hexagon = $6 \times \frac{\sqrt{3}}{4} \cdot \left(\frac{h}{\sqrt{3}}\right)^2 = \frac{3\sqrt{3}}{2} \cdot \frac{A}{\pi}$ metre²

26.18 **(3)**



Clearly $\angle OPA = \angle OPC = \frac{\pi}{4}$

Let $OP = h$

So $\frac{OA}{OP} = \tan 45° = 1$ $\therefore$ $OA = h$ and $AC = 2h$

Also $\frac{OA}{PA} = \cos\frac{\pi}{4}$ $\Rightarrow$ $PA = \sqrt{2}\,h$

Also $AC = \sqrt{2}\,AB$ $\Rightarrow$ $2h = \sqrt{2}\,AB$

i.e. $AB = \sqrt{2}\,h$ $\therefore$ $PA = AB = PB$ (Similarly)

So ΔBPA is equilateral triangle

Hence required angle = 60°

26.19 **(1)**

We have $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

or $b^2 - (2c\cos A)\,b + (c^2 - a^2) = 0$, which is quadratic equation in b

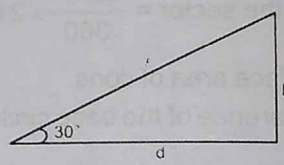
$\because$ $c \sin A < a < c$

$\therefore$ two triangles will be obtained but this is possible when two values of third side are also obtained clearly two values of side b will be b_1 and b_2.

let these are roots of above equation

$\therefore$ Sum of roots $= b_1 + b_2 = 2c\cos A$

26.20 **(3)**



$H = 20 = \ell + h,\ \ell = \frac{d}{\cos 30°}$

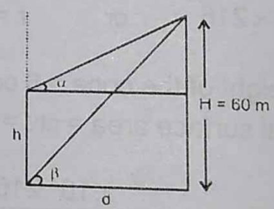
$h = d \tan 30°$

$\therefore$ $d = \frac{20}{\sec 30° + \tan 30°} = \frac{20}{\sqrt{3}}$

and $h = d\tan 30° = \frac{20}{3}$

26.21 **(2)**

$H = d\tan\beta$ and $H - h = d\tan\alpha$



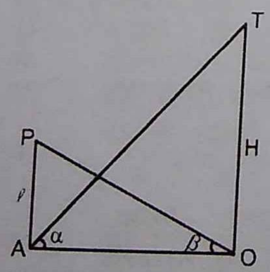
$\Rightarrow$ $\frac{60}{60-h} = \frac{\tan\beta}{\tan\alpha} \Rightarrow -h = \frac{60\tan\alpha - 60\tan\beta}{\tan\beta}$

$\Rightarrow$ $h = \frac{60\sin(\beta-\alpha)}{\cos\alpha\cos\beta.\frac{\sin\beta}{\cos\beta}}$

$\Rightarrow$ $x = \cos\alpha\,\sin\beta$

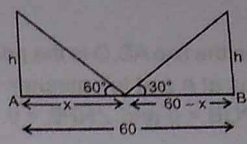
26.22 **(2)**

From figure, we can deduce



$H = \ell \tan\alpha \cot\beta$

26.23 (1)



$\tan 60° = \dfrac{h}{x} \quad \Rightarrow \quad \dfrac{\sqrt{3}}{1} = \dfrac{h}{x}$

$\Rightarrow \quad h = \sqrt{3}\,x \qquad$ .... (i)

$\tan 30° = \dfrac{h}{60 - x} \quad \Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{h}{60 - x}$

$\Rightarrow \quad 60 - x = \sqrt{3}\,h \qquad$ .... (ii)

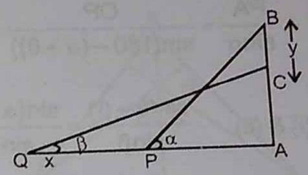
From (i) & (2),

$60 - x = \sqrt{3}(\sqrt{3}\,x)$

$\dfrac{60}{4} = x = 15$

Then $h = \sqrt{3}\,x = 15\sqrt{3}$ m

26.24 (1)



PB = QC = ℓ (length of ladder)

$\Rightarrow \quad PA = \ell \cos\alpha,\ AB = \ell \sin\alpha$

$\Rightarrow \quad AC = \ell \sin\beta,\ QA = \ell \cos\beta$

$\Rightarrow \quad CB = AB - AC = \ell(\sin\alpha - \sin\beta)$

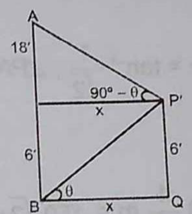
$\Rightarrow \quad y = \ell(\sin\alpha - \sin\beta)$

and $\quad QP = x = AQ - AP = \ell(\cos\beta - \cos\alpha)$

$$\Rightarrow \quad \frac{CB}{QP} = \frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \frac{y}{x} = \frac{2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}}{2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}} \quad \Rightarrow \quad \frac{y}{x} = \cot\frac{\alpha+\beta}{2}$$

$$\Rightarrow \quad x = y\tan\left(\frac{\alpha+\beta}{2}\right)$$

26.25 (3)



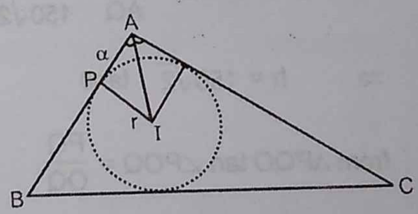
$\tan\theta = \dfrac{6}{x},\ \tan(90 - \theta) = \dfrac{18}{x}$

$\therefore \quad \dfrac{6}{x}.\dfrac{18}{x} = 1$

$\Rightarrow \quad x^2 = 6 \times 18$

$\Rightarrow \quad x = 6\sqrt{3}$

26.26 (2)



Let AI is angle bisector of ∠BAC

$\dfrac{r}{\alpha} = \tan\dfrac{A}{2} \qquad \therefore \qquad \alpha = r\cot\dfrac{A}{2}$

Similarly $\beta = r\cot\dfrac{B}{2}$; $\gamma = r\cot\dfrac{C}{2}$

In a ΔABC, we have the identity

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$$

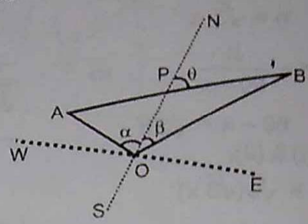
$$\therefore \quad \frac{\alpha}{r} + \frac{\beta}{r} + \frac{\gamma}{r} = \frac{\alpha}{r}.\frac{\beta}{r}.\frac{\gamma}{r} \quad \Rightarrow \quad \frac{1}{r}(\alpha+\beta+\gamma) = \frac{1}{r^3}\alpha\beta\gamma$$

$$\Rightarrow \quad r = \sqrt{\frac{\alpha\beta\gamma}{\alpha+\beta+\gamma}}$$

26.27 **(3)**

The car is moving on the line AB.O is the point of observation. At the time of observation the car is at P. Ten minutes earlier it was at A and ten minutes afterwards it is at B. So AP = PB

Also $\angle POA = \alpha$, $\angle POB = \beta$ and $\angle NPB = \theta$.



In the ΔOPB, $\dfrac{PB}{\sin\beta} = \dfrac{OP}{\sin\angle OBP}$

or $\dfrac{PB}{\sin\beta} = \dfrac{OP}{\sin(\theta-\beta)}$ .... (i)

in the ΔOPA, $\dfrac{PA}{\sin\alpha} = \dfrac{OP}{\sin OAP}$

or $\dfrac{PA}{\sin\alpha} = \dfrac{OP}{\sin(180-(\alpha+\theta))}$ .... (ii)

From (i) & (ii) $\dfrac{\sin(\theta-\beta)}{\sin\beta} = \dfrac{\sin(\alpha+\theta)}{\sin\alpha}$ ($\because$ PA = PB)

$\Rightarrow$ $2\cos\theta = \sin\theta(\cot\beta - \cot\alpha)$ $\Rightarrow$ $\tan\theta = \dfrac{2}{\cot\beta - \cot\alpha}$

26.28 **(2)**

Let the height of tower PQ = h m.

Then $\angle POQ = 30°$, $\angle QOA = \alpha = \tan^{-1}\dfrac{1}{\sqrt{2}}$, $\angle PAQ = \theta$

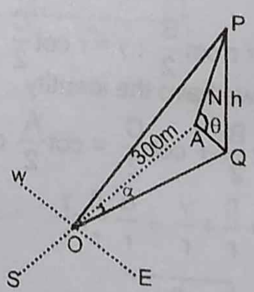
from the right angled ΔQAO

$\dfrac{AQ}{OA} = \tan\alpha$ $\therefore$ $AQ = 300\dfrac{1}{\sqrt{2}}\,m = 150\sqrt{2}\,m.$

from ΔPQA, $\tan\theta = \dfrac{PQ}{AQ} = \dfrac{h}{150\sqrt{2}}$

$\Rightarrow$ $h = 150\sqrt{2}\,.\,\tan\theta$ ... (i)

from ΔPQO $\tan\angle POQ = \dfrac{PQ}{OQ}$



$\Rightarrow$ $\tan 30° = \dfrac{h}{\sqrt{OA^2 + AQ^2}}$

$\Rightarrow$ $\dfrac{1}{\sqrt{3}} = \dfrac{h}{\sqrt{300^2 + (150\sqrt{2})^2}}$

$\Rightarrow$ $h = 150\sqrt{2}$

$\Rightarrow$ $150\sqrt{2} = 150\sqrt{2}\cdot\tan\theta$

$\Rightarrow$ $\tan\theta = 1$ $\Rightarrow$ $\theta = 45°$

26.29 (1)

$\left(\frac{b+a+c}{b}\right)\left(\frac{c-a+b}{c}\right) = k$ (let)

$\Rightarrow \quad (b + c)^2 - a^2 = kbc$

$\Rightarrow \quad b^2 + c^2 - a^2 = (k - 2)\,bc$

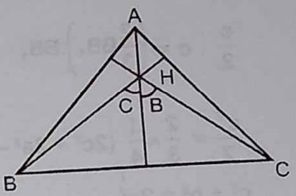
$\Rightarrow \quad \frac{b^2+c^2-a^2}{2bc} = \frac{k-2}{2} \quad$ or $\quad \cos A = \frac{k-2}{2}$

$\because \quad -1 \le \cos A \le 1 \quad \Rightarrow \quad -1 \le \frac{k-2}{2} \le 1$

$\Rightarrow \quad k \in [0, 4]$

26.30 (1)

In ΔHBC is we apply Sine-rule, then we get



$\frac{BC}{\sin(B+C)} = 2R'$

$\frac{a}{\sin A} = 2R' \Rightarrow 2R = 2R' \Rightarrow R = R'$

$\therefore$ circumradius of ΔHBC (i.e. R') = R

Similarly we can prove for ΔHCA and ΔHAB.

26.31 (1)

$f = R\cos A$, $g = R\cos B$, $h = R\cos C$.

$\because \quad \frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \frac{2R\sin A}{R\cos A} + \frac{2R\sin B}{R\cos B} + \frac{2R\sin C}{R\cos C}$

$= 2\left(\sum \tan A\right)$

$\because \quad \frac{abc}{fgh} = 8\left(\prod \tan A\right)$

$\because \quad \frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \lambda \frac{abc}{fgh}$

$\Rightarrow 2\sum \tan A = \lambda.8\left(\prod \tan A\right) \quad \Rightarrow \quad \lambda = \frac{1}{4}$

26.32 (4)

$\because \quad a : b : c = 4 : 5 : 6$

$\therefore \quad a = 4k,\ b = 5k\ ,\ c = 6k$

$\because \quad \cos B = \frac{c^2+a^2-b^2}{2ac} = \frac{9}{16}$

$\because \quad \cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{25+36-16}{2\times5\times6} = \frac{3}{4}$

$\because \quad \cos 3A = 4\cos^3 A - 3\cos A$

$= 4 \times \frac{27}{64} - 3 \times \frac{3}{4} = \frac{27}{16} - \frac{9}{4}$

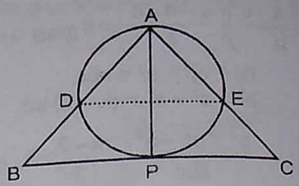
$= \frac{27-36}{16} = \frac{-9}{16}$

$\cos 3A = -\cos B = \cos(\pi - B)$

$\therefore \quad 3A + B = \pi$

26.33 (4)

$\because \dfrac{DE}{\sin A} = AP$



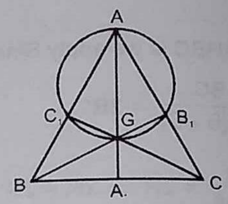
$\Rightarrow DE = \dfrac{2\Delta}{a} \sin A$

$= \dfrac{2\Delta \sin A}{2R \sin A} = \dfrac{\Delta}{R}$

26.34 (3)

$\because$ A, C_1, G and B_1 are cyclic

$\therefore$ $BC_1 \cdot BA = BG \cdot BB_1$



$\dfrac{c}{2} \cdot c = \left(\dfrac{2}{3}BB_1\right).BB_1$

$\dfrac{c^2}{2} = \dfrac{2}{3} \times \dfrac{1}{4}\ (2c^2 + 2a^2 - b^2)$

$\Rightarrow \quad c^2 + b^2 = 2a^2$

26.35 (1)

$\cos B \cdot \cos C + \sin B \cdot \sin C \geq 1$

because $\sin B \cdot \sin C \cdot \sin^2 A$ is positive and $\sin^2 A \leq 1$.

or $\quad \cos(B - C) \geq 1$.

But $\cos(B - C) > 1$. So $\cos(B - C) = 1$. Therefore

$B = C$ and then $\sin A = 1$

$\therefore \quad A = \dfrac{\pi}{2} : B = C = \dfrac{\pi}{4}$

26.36 (2)

$\cot A \cdot \cot B \cdot \cot C > 0$

$\Rightarrow \quad \cot A > 0, \cot B > 0, \cot C > 0$

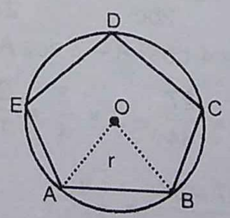
because two or more of cotA, cotB, cotC can not be negative at the same time in a triangle

26.37 (4)

$OA = OB = r$ and $\angle AOB = \dfrac{360^\circ}{5} = 72^\circ$

$\therefore \quad$ area $(\Delta AOB) = \dfrac{1}{2} \cdot r \cdot r \cdot \sin 72^\circ = \dfrac{1}{2} r^2 \cos 18^\circ$

area (pentagon) $= \dfrac{5}{2} r^2 \cos 18^\circ$



$A_1/A_2 = \dfrac{2\pi}{5} \sec \dfrac{\pi}{10}$

So $\quad$ A is wrong. & R is correct

26.38 (2)

Statement-1 : $\because$ AM $\geq$ HM

$$\Rightarrow \frac{r_1+r_2+r_3}{3} \geq \frac{3}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}$$

$$\Rightarrow \frac{r_1+r_2+r_3}{3} \geq \frac{3}{\left(\frac{1}{r}\right)} \qquad \because \frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r}$$

$$\Rightarrow \frac{r_1+r_2+r_3}{r} \geq 9 \qquad \text{(correct)}$$

Statement-2 : $\because$ if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$$\Rightarrow \frac{\cos A}{2R\sin A} = \frac{\cos B}{2R\sin B} = \frac{\cos C}{2R\sin C}$$

$\Rightarrow$ cot A = cot B = cot C

$\Rightarrow$ A = B = C $\Rightarrow$ ΔABC is equilateral

$\therefore$ $r_1 = r_2 = r_3$

$$\therefore \frac{r_1+r_2+r_3}{r} = \frac{3r_1}{r} = 3\cot\frac{A}{2}\cot\frac{C}{2}$$

$$= 3\cot^2\frac{A}{2} = 3\cot^2\frac{\pi}{6} = 3 \times 3 = 9 \qquad \text{(correct)}$$

But statement-2 does not explain statement-1

26.39 (3)

Statement-1 :

$$\because \text{ H.M. of the three ex-radii} = \frac{3}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \frac{3\Delta}{s-a+s-b+s-c} = \frac{3\Delta}{s} = 3r$$

= 3 times the inradius

$\therefore$ statement-1 is true

Statement-2 : $\because$ L.H.S. = $r_1 + r_2 + r_3$

$$= \frac{\Delta}{s-a} + \frac{\Delta}{s-b} + \frac{\Delta}{s-c}$$

$$= \Delta\left[\frac{(s-b)(s-c)+(s-a)(s-c)+(s-a)(s-b)}{(s-a)(s-b)(s-c)}\right]$$

$$= s\Delta\left[\frac{3s^2-2s(a+b+c)+ab+bc+ca}{\Delta^2}\right]$$

$$= \frac{s\Delta\left[ab+bc+ca-s^2\right]}{\Delta^2}$$

$$= \frac{s\left(ab+bc+ca-s^2\right)}{\Delta}$$

$$\because \text{ R.H.S.} = 4R = \frac{abc}{\Delta}$$

$\therefore$ L.H.S. $\neq$ R.H.S.

$\therefore$ Statement 2 is false.

27. Inverse Trigonometric Function

27.1 **(3)**

$2x - x^2 = -(x^2 - 2x + 1) + 1 = -(x-1)^2 + 1 \le 1$

$\therefore$ range of $2x - x^2$ is $(-\infty, 1]$

$\therefore$ range of $\sec^{-1}(2x - x^2)$ is $\left(\frac{\pi}{2}, \pi\right] \cup \{0\}$

27.2 **(3)**

$3\pi < 10 < 3\pi + \frac{\pi}{2}$

$\therefore$ $\sin^{-1}(\sin 10) = 3\pi - 10$

$\therefore$ $3\pi - 10 < x^2 - 6x - 1 + 3\pi \Rightarrow (x-3)^2 > 0 \Rightarrow x \in R - \{3\}$

27.3 **(3)**

$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$, $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 150°$

27.4 **(4)**

Clearly, $\sec(\sec^{-1}x)$ is defined as x when $x \le -1$ or $x \ge 1$

27.5 **(4)**

Clearly, $\tan^{-1}2 + \tan^{-1}3 = \pi + \tan^{-1}\left(\frac{2+3}{1-2(3)}\right) = \pi + \tan^{-1}(-1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

27.6 **(2)**

Clearly $\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\frac{\sqrt{1-\left(\frac{4}{5}\right)^2}}{\frac{4}{5}} = \tan^{-1}\frac{3}{4}$

and $\cot^{-1}\left(\frac{3}{2}\right) = \tan^{-1}\left(\frac{2}{3}\right)$

Now $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3} = \tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}} = \tan^{-1}\frac{17}{6}$

$\therefore$ ans. is $\tan\tan^{-1}\frac{17}{6} = \frac{17}{6}$

27.7 (3)

Given series $= \tan^{-1}\frac{(x+1)-x}{1+x(x+1)} + \tan^{-1}\frac{(x+2)-(x+1)}{1+(x+1)(x+2)} + \ldots\ldots$ n terms

$= \tan^{-1}(x+1) - \tan^{-1}x + \tan^{-1}(x+2) - \tan^{-1}(x+1) + \ldots\ldots$ n terms

$= \tan^{-1}(x+n) - \tan^{-1}x$

27.8 (4)

Given $= \tan^{-1}1 + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots\ldots$ n terms

$= \tan^{-1}1 + \tan^{-1}\frac{2-1}{1+1(2)} + \tan^{-1}\frac{3-2}{1+3(2)} + \tan^{-1}\frac{4-3}{1+4(3)} + \ldots\ldots$ n terms

$= \tan^{-1}1 + \tan^{-1}2 - \tan^{-1}1 + \tan^{-1}3 - \tan^{-1}2 + \tan^{-1}4 - \tan^{-1}3 + \ldots\ldots$ n terms $= \tan^{-1}n$

27.9 (3)

$\tan^{-1}2 = \frac{\pi}{2} - \cos^{-1}x \Rightarrow \sin^{-1}\frac{2}{\sqrt{5}} = \sin^{-1}x \Rightarrow x = \frac{2}{\sqrt{5}}$

27.10 (1)

Clearly $x = -1$

$\therefore \quad \mathrm{cosec}^{-1}(-1) = -\frac{\pi}{2}$

27.11 (1)

Clearly, $\sin^{-1}(\sin x) = \sin^{-1}\sin(x - 2\pi)$ where $x - 2\pi \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \quad \sin^{-1}(\sin x) = x - 2\pi.$

27.12 (3)

$2\tan^{-1}x + \frac{\pi}{2} = \pi$

$\therefore \quad \tan^{-1}x = \frac{\pi}{4}$

$\Rightarrow \quad x = 1$

27.13 (1)

Clearly $\sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2}$ (each)

$\therefore \quad x = y = z = 1$

27.14 (3)

$\sin\cot^{-1}x = \frac{1}{\sqrt{1+x^2}}$ and $\cos\tan^{-1}\frac{1}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+\frac{1}{1+x^2}}} = \sqrt{\frac{x^2+1}{x^2+2}}$

27.15 (2)

$\sin^{-1}\frac{4}{5} = \cos^{-1}\sqrt{1-\left(\frac{4}{5}\right)^2} = \cos^{-1}\frac{3}{5}$

$\therefore$ given expression $= \cos\left(\cos^{-1}\frac{3}{5}+\cos^{-1}\frac{2}{3}\right) = \frac{3}{5}\times\frac{2}{3} - \frac{4}{5}\times\frac{\sqrt{5}}{3} = \frac{6-4\sqrt{5}}{15}$

27.16 (2)

$\cos^{-1}(\cos x) = x$ as $x \in \left(\frac{\pi}{2}, \pi\right)$

$\sin^{-1}(\sin x) = \pi - x$

$\therefore$ given expression $= \sin^{-1}[\cos(x + \pi - x)] = \sin^{-1}(\cos\pi) = \sin^{-1}(-1) = -\frac{\pi}{2}$

27.17 (1)

Given expression $= \tan^{-1}2 + \tan^{-1}\frac{2}{9} + \tan^{-1}\frac{2}{25} + \tan\frac{2}{49} + \ldots\ldots\ldots$

General term $= \frac{2}{(2n-1)^2} = \frac{2}{4n^2-4n+1} = \frac{2}{1+4n(n-1)} = \frac{2n-(2n-2)}{1+2n(2n-2)}$

$T_n = \tan^{-1}2n - \tan^{-1}(2n-2)$

$\therefore$ sum of series $= \tan^{-1}2 - \tan^{-1}0 + \tan^{-1}4 - \tan^{-1}2 + \tan^{-1}6 - \tan^{-1}4 + \ldots.$ upto n terms $= \tan^{-1}2n$

27.18 (3)

$\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right) \qquad x \neq 0$

let $\theta = \frac{1}{2}\cos^{-1}x \qquad 2\theta \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$= \tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)$

$= \frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta} = 2\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$

$= \frac{2}{\cos 2\theta} = \frac{2}{\cos\cos^{-1}x} = \frac{2}{x}$

27.19 (4)

$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = 4^\circ \qquad x \neq 0$

taking tan on both side

$\frac{\sqrt{1+x^2}-1}{x} = \tan 4^\circ$

$\sqrt{1+x^2} = 1 + x\tan 4^\circ$

$1 + x^2 = 2x\tan 4^\circ + 1 + x^2\tan^2 4^\circ$

$x = 0, \frac{2\tan 4^\circ}{1-\tan^2 4} \qquad$ since $x \neq 0$

Ans. $x = \tan 8^\circ$

27.20 (2)

$\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6} \Rightarrow -\infty < \frac{n}{\pi} < \cot\frac{\pi}{6}$

$\Rightarrow n < \pi\sqrt{3} \quad n \in N, \quad n_{max} = 5$

27.21 (2)

$\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right) = \frac{\pi}{2}$

Taking sin on both side

$\frac{3\sin 2\theta}{5+4\cos 2\theta} = 1$

$3\sin 2\theta = 5 + 4\cos 2\theta$

$\frac{6\tan\theta}{1+\tan^2\theta} = 5 + 4\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\tan^2\theta - 6\tan\theta + 9 = 0$

$\tan\theta = 3$

27.22 (3)

$$\sqrt{1+x^2}\left[\left\{x\cos\cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)+\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}^2-1\right]^{1/2}$$

$$= \sqrt{1+x^2}\left[\left(\frac{x^2}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right)^2-1\right]^{\frac{1}{2}}$$

$= \sqrt{1+x^2}\,.\,x$ Hence (3) is correct.

27.23 (1)

Clearly $\alpha = 2, \beta = -\frac{1}{2}$

27.24 (2)

$\tan(\tan^{-1}x + \tan^{-1}y + \tan^{-1}z) = \frac{x+y+z-xyz}{1-(xy+yz+zx)}$

27.25 (4)

$\tan^2(\sec^{-1}2) + \cot^2(\mathrm{cosec}^{-1}3)$

$\tan^2(\tan^{-1}\sqrt{3}) + \cot^2(\cot^{-1}\sqrt{8})$

$3 + 8 = 11$

27.26 (2)

Using properties

$\therefore \quad \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \Rightarrow \frac{a}{x} = \frac{x}{b} \Rightarrow x = \sqrt{ab}$

statement-1 is true

$$\tan^{-1}\left(\frac{m}{n}\right) + \tan^{-1}\left(\frac{1-\frac{m}{n}}{1+\frac{m}{n}}\right) = \tan^{-1}\frac{m}{n} + \tan^{-1}1 - \tan^{-1}\frac{m}{n} = \frac{\pi}{4}$$

28. STATISTICS

28.1 **(3)**

Mode of the data is 8 as it is repeated maximum number of times.

28.2 **(2)**

Corrected $\sum x = 40 \times 200 - 50 + 40 = 7990$

corrected $\bar{x} = \frac{7990}{200} = 39.95$

Incorrect $\sum x^2 = n(\sigma^2 + \bar{x}^2) = 200\,(5^2 + 40^2) = 365000$

correct $\sum x^2 = 365000 - 2500 + 1600 = 364100$

corrected $\sigma = \sqrt{\frac{364100}{200} - (39.95)^2} = \sqrt{(1820.5 - 1596)}$

$= \sqrt{224.5}$

$= 14.98$

28.3 **(1)**

$$\mu_1{}^1 = \frac{\sum_{r=0}^{n} r.\,{}^nC_r}{\sum_{r=0}^{n} {}^nC_r} = \frac{n\sum_{r=1}^{n} {}^{n-1}C_{r-1}}{2^n} = \frac{n.2^{n-1}}{2^n} = \frac{n}{2}$$

$$\mu_2{}^1 = \frac{\sum_{r=0}^{n} r^2.\,{}^nC_r}{\sum_{r=0}^{n} {}^nC_r} = \frac{1}{2^n}\sum_{r=0}^{n}\{r(r-1)+r\}.\,{}^nC_r$$

$$= \frac{1}{2^n}\left\{n(n-1)\sum_{r=2}^{n} {}^{n-2}C_{r-2} + n\sum_{r=1}^{n} {}^{n-1}C_{r-1}\right\}$$

$$= \frac{1}{2^n}\,[n.(n-1).2^{n-2} + n.2^{n-1}]$$

$$= \frac{n(n-1)}{4} + \frac{n}{2}$$

Now variance $\mu = \mu_2{}^1 - (\mu_1{}^1)^2$

$$= \frac{n(n-1)}{4} + \frac{n}{2} - \left(\frac{n}{2}\right)^2 = \frac{n}{4}$$

28.4 (1)
Let n_1 = number of mail imployees
n_2, = number of female employees
$\bar{x}_1$ = average salary of male employees
$\bar{x}_2$ = average salary of female employees

then $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$$500 = \frac{520n_1 + 420n_2}{n_1 + n_2}$$

$20n_1 = 80n_2$

$n_1 : n_2 = 4 : 1$

$\therefore$ percentage of male employees = 80
& that of female employees = 20

28.5 (3)

$\bar{x} = \dfrac{\Sigma x_i}{n}, \Sigma x_i = n\bar{x}$

New mean $= \dfrac{\Sigma\lambda x_i}{n} = \lambda\dfrac{\Sigma x_i}{n} = \lambda\bar{x}$.

28.6 (1)
It is obvious.

28.7 (3)
It is a fundamental property.

28.8 (2)

$$\text{Weighted mean} = \frac{1.1^2 + 2.2^2 + \ldots\ldots + n.n^2}{1^2 + 2^2 + \ldots\ldots.. + n^2} = \frac{\Sigma n^3}{\Sigma n^2} = \frac{\frac{n(n+1)}{2}\frac{n(n+1)}{2}}{\frac{n(n+1)(2n+1)}{6}} = \frac{3n(n+1)}{2(2n+1)}$$

28.9 (1)
Marks obtained from 3 subjects out of 300
= 75 + 80 + 85 = 240
If the marks of another subject is added, then the marks will be $\geq$ 240 out of 400

$\therefore$ Minimum average marks $= \dfrac{240}{4} = 60\%$, [When marks in the fourth subject = 0]

28.10 (2)

Also $\bar{x} = 4 \Rightarrow \frac{x_1 + x_2 + 1 + 2 + 6}{5} = 4$

$\Rightarrow$ $x_1 + x_2 = 11$ ..... (ii)

(i), (ii) $\Rightarrow x_1, x_2 = 4, 7$

28.33 (1)

$\because$ $\frac{1}{n}\Sigma(x_i + 2)^2 = 18$ and $\frac{1}{n}\Sigma(x_i - 2)^2 = 10$

$\Rightarrow$ $\Sigma(x_i + 2)^2 = 18n$ and $\Sigma(x_i - 2)^2 = 10n$

$\Rightarrow$ $\Sigma(x_i + 2)^2 + \Sigma(x_i - 2)^2 = 28\,n$ and $\Sigma(x_i + 2)^2 - \Sigma(x_i - 2)^2 = 8n$

$\Rightarrow$ $2\Sigma x_i^2 + 8n = 28\,n$ and $8\,\Sigma x_i = 8n$

$\Rightarrow$ $\Sigma x_i^2 = 10\,n$ and $\Sigma x_i = n$

$\Rightarrow$ $\frac{\Sigma x_i^2}{n} = 10$ and $\frac{\Sigma x_i}{n} = 1$ $\therefore$ $\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} = \sqrt{10 - (1)^2} = 3$

28.34 (1)

$$\text{Weighted mean} = \frac{1.1^2 + 2.2^2 + ... + n.n^2}{1^2 + 2^2 + ... + n^2}$$

$$= \frac{\Sigma n^3}{\Sigma n^2} = \frac{3n(n+1)}{2(2n+1)}$$

28.35 (1)

$N = \Sigma f_i = k[{}^nC_0 + {}^nC_1 + ...+{}^nC_n] = k(1 + 1)^n = k.2^n$

$\Sigma f_i x_i = k[1.\ {}^nC_1 + 2.\ {}^nC_2 + ...+ n.\ {}^nC_n]$

$= kn\ 2^{n-1}$

$\therefore \quad \bar{x} = \frac{1}{2^n} n.2^{n-1} = \frac{n}{2}$

28.36 (1)

$$\text{Mean} = \frac{1+3+5+........+(2n-1)}{n} = \frac{n^2}{n} = n$$

28.37 (3)

$$2^2 + 4^2 + 6^2 + ... + (2n)^2 = 4\ (1^2 + 2^2 + ... + n^2) = 4\ .\ \frac{n(n+1)(2n+1)}{6}$$

$$2 + 4 + 6 + ... + 2n \text{ or } 2(1 + 2 + ... + n) = n(n + 1)$$

$$\therefore \quad \text{variance} = \frac{2}{3}(n+1)(2n+1) - (n+1)^2 = \frac{(n+1)(4n+2-3n-3)}{3} = \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$$

$\therefore$ Statement-1 is false

Statement-2 is true

29. MATHEMATICAL REASONING

29.1 **(3)**

p : We control population, q : we prosper

$\therefore$ we have $p \Rightarrow q$

Its negation is $\sim (p \Rightarrow q)$ i.e. $p \wedge \sim q$.

i.e. we control population but we do not prosper.

29.2 **(2)**

$(p \Rightarrow q) \wedge (q \Rightarrow p)$ means (माध्य) $p \Leftrightarrow q$

29.3 **(4)**

Obiviously $\sim p \wedge q$

$= \sim (q \to p)$

29.4 **(2)**

$p \Rightarrow q$ is logically equivalent to $\sim q \Rightarrow \sim p$

$\therefore (p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tatutology but not a contradiction

$\therefore (p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tatutology

29.5 **(2)**

$(p \vee q) \wedge \sim p = (p \wedge \sim p) \vee (q \wedge \sim p) = f \vee (q \wedge \sim p) = q \wedge \sim p = \sim p \wedge q$

29.6 **(1)**

p : Mumbai is in England

q : 2 + 2 = 5

If p, then q $\Rightarrow$ $p \to q$ $\Rightarrow$ $F \to F$ $\Rightarrow$ T

29.7 **(1)**

$\sim (p \to q) \equiv (p \wedge \sim q)$

29.8 **(1)**

Converse of $p \to q$ is $q \to p$.

29.9 **(4)**

Contrapositive of $p \to q$ is $\sim q \to \sim p$.

29.10 **(1)**

Converse of $p \to q$ is $q \to p$.

29.11 **(1)**

$p \to (q \to r) \equiv \sim p \vee (q \to r) \equiv \sim p \vee (\sim q \vee r) \equiv [(\sim p \vee (\sim q)] \vee r \equiv \sim (p \wedge q) \vee r \equiv p \wedge q \to r$

$\therefore$ given statement is a tautology.

29.12 **(2)**

Let p : Paris is in France, q : London is in England

$\therefore$ we have $p \wedge q$

Its negation is $\sim (p \wedge q) = \sim p \vee \sim q$

i.e. Paris is not in France or London is not in England.

29.13 **(1)**

p	q	p ^ q	(p ^ q) ⇒ p
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

∴ $(p \wedge q) \Rightarrow p$ is a tautology.

29.14 **(2)**

$(p \wedge q) \wedge (q \wedge r)$ is true means $p \wedge q$, $q \wedge r$ are both true.

⇒ p, q, r are all true.

29.15 **(1)**

Correct results is $(\sim p \vee \sim q) \Rightarrow (r \wedge s)$

So, $\sim (p \wedge q) \Rightarrow (r \wedge s)$.

29.16 **(1)**

$\sim (p \Leftrightarrow q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

29.17 **(2)**

Interchange v, ∧ and t, f

29.18 **(1)**

$S_1 \wedge S_2 \wedge S_3 \rightarrow S$ is a tautology means when S_1, S_2, S_3 are all true, then S is true. So argument is valid.

29.19 **(1)**

p : it is a good watch

q : it is a Titan watch

$S_1 : p \rightarrow q$

$S_2 : q$

$S : p$

p	q	p → q	S_1	S_2	S
T	F	F	F	F	T
F	T	T	T	T	F
T	T	T	T	T	T
F	F	T	T	F	F

As $S_1 : T$, $S_2 : T \not\Rightarrow S : T$ so invalid argument.

29.20 **(1)**

$(p \rightarrow q) \rightarrow r \equiv (\sim p \vee q) \rightarrow r \equiv \sim(\sim p \vee q) \vee r \equiv (p \wedge \sim q) \vee r$

Its dual $= (p \vee \sim q) \wedge r = (q \rightarrow p) \wedge r$.

29.21 **(3)**

p	q	q → p	p → (q → p)	p → (q ↔ p)	p → (p → q)	p → (p ∨ q)
T	F	T	T	F	F	T
F	T	F	T	T	T	T
T	T	T	T	T	T	T
F	F	T	T	T	T	T

29.22 **(1)**

Contrapositive of $p \to q$ is $\sim q \to \sim p$.

29.23 **(3)**

Negation of $7 > 9$ is 7 is not greater than 9.

29.24 **(1)**

Either $x > 1$ or $x < 1$ $\Rightarrow$ $x \neq 1$

29.25 **(3)**

p	q	~ p	~ q	~ p ↔ q	q ∨ p	(p ↔ ~ q)	~ (p ↔ ~ q)
T	T	F	F	F	T	F	T
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	F
F	F	T	T	F	F	F	T

29.26 **(4)**

(1) "2 + 2 = 5" is false

(2) "Sun is a star" is scientific truth

(3) "There are 40 days in a month" is always false

Hence (4) is correct answer.

29.27 **(4)**

Negation of p means not happening of p.

29.28 **(1)**

If p, then q

$\Rightarrow$ q is necessary for p and p is sufficient for q.

29.30 **(2)**

Since $\sim r$ is true, therefore, r is false.

Again $q \to r$ is true and r is false, therefore, q has to be false.

($\because$ A true statement cannot lead to a false statement)

Further $p \to q$ is true and q is false, therefore, p is also false.

29.31 **(3)**

The statements given in (1), (2) and (4) are logical statements. The statement (1) is false, (2) is false and (4) is true.

The sentence in (3) is not logically true or false. Manisha may look beautiful to some persons and may not look beautiful to others.

29.32 **(4)**

p	q	$\sim p$	$\sim p \vee q$	$p \rightarrow (\sim p \vee q)$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore$ If $p \rightarrow (\sim p \vee q)$ is false p is true and q is false

29.33 **(1)**

Statement-2 is correct definition of inclusive OR.

29.34 **(4)**

$(p \wedge \sim q) \vee q = (p \vee q) \wedge (\sim q \vee q) = (p \vee q) \wedge t = p \vee q$

SECTION-II
PRACTICE TEST PAPERS

SOLUTIONS

SOLUTION TO PART TEST-1 TOPIC : COORDINATE GEOMETRY-2D (XI)

Answers

1.	(3)	2.	(2)	3.	(4)	4.	(4)	5.	(2)	6.	(2)	7.	(1)
8.	(3)	9.	(2)	10.	(3)	11.	(1)	12.	(4)	13.	(4)	14.	(2)
15.	(4)	16.	(2)	17.	(4)	18.	(1)	19.	(4)	20.	(1)	21.	(3)
22.	(2)	23.	(2)	24.	(3)	25.	(3)	26.	(1)	27.	(1)	28.	(3)
29.	(1)	30.	(4)										

SOLUTIONS

1. Orthocentre of the triangle is the point of intersection of the lines

$x + y - 1 = 0$ and $x - y + 3 = 0$

i.e. $(-1, 2)$

2. $x \cos \alpha + y \sin \alpha = p \quad \Rightarrow \quad x \cos 15° + y \sin 15° = 4$

$\Rightarrow \quad x\frac{(\sqrt{3}+1)}{2\sqrt{2}} + y\frac{(\sqrt{3}-1)}{2\sqrt{2}} = 4 \quad \Rightarrow \quad (\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$

3. Equation of the pair of st. line is

$$x^2 + \frac{y^2}{4} - \left(\frac{y-3x}{c}\right)^2 = 0$$

$4c^2x^2 + c^2y^2 - 4(y^2 + 9x^2 - 6xy) = 0$

since the lines are perpendicular to each other

$\therefore \quad 5c^2 - 4 - 36 = 0$ i.e. $c^2 = 8$

$\therefore \quad$ sum of the values of c is 0

4. The point $\left(3+\frac{3}{\sqrt{2}}; \frac{3}{\sqrt{2}}\right)$ does not satisfy the given circle in (1) and (3).

The centre of the circle given in B is $\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ which does not lie on $x - y = 3$.

From option (4) ; $\left(3+\frac{3}{\sqrt{2}}-3\right)^2 + \left(\frac{3}{\sqrt{2}}-0\right)^2 = \frac{9}{2} + \frac{9}{2} = 9$

5. center $(2, 4)$, radius = 4

Image of $(2, 4)$ in $y = x$ is $(4, 2)$

$\therefore$ equation of image is $(x-4)^2 + (y-2)^2 = 16$

$\Rightarrow \quad x^2 + y^2 - 8x - 4y + 4 = 0$

6. The x coordinate of P on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is 8.

$\Rightarrow \quad 16y^2 = 432$

$y = \sqrt{27}$

$\therefore \quad P\left(8, \sqrt{27}\right)$

$e = \frac{5}{4}$

The reflected ray passes through the other focus (– 5, 0).

∴ the equation of the reflected ray is

$$y - \sqrt{27} = \frac{0-\sqrt{27}}{-5-8}(x-8)$$

$$\Rightarrow \quad 3\sqrt{3}\,x - 13y + 15\sqrt{3} = 0.$$

7. $a > 0$ and $a^2 - 5 > 0 \Rightarrow a > \sqrt{5}$ ....(i)

Since y-axis is major axis

$\Rightarrow 4a > a^2 - 5$

$\Rightarrow a^2 - 4a - 5 < 0$

$\Rightarrow a \in (-1, 5)$ ....(ii)

by (i) $\cap$ (ii)

$\Rightarrow a \in (\sqrt{5}, 5)$

8. $y^2 - 4y = 8x - 2$

$y^2 - 4y + 4 = 8x + 2$

$(y-2)^2 = 8\left(x + \frac{1}{4}\right)$

$Y^2 = 8X$

where $X = x + \frac{1}{4}$

$Y = y - 2$

for $Y^2 = 8X$, $a = 2$

$$L = 4a\,\text{cosec}^2\theta = 4 \times 2 \times \text{cosec}^2 15^\circ = \frac{8}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2} = \frac{64}{4-2\sqrt{3}} = 32\,(2+\sqrt{3})$$

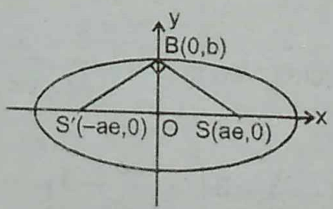
9. $\frac{b}{-ae} \times \frac{b}{ae} = -1$

$b^2 = a^2e^2$

$a^2(1 - e^2) = a^2e^2$

$2e^2 = 1$

$e = \frac{1}{\sqrt{2}}$



10. Other asymptote will be perpendicular to $3x - 4y - 6 = 0$

Let it is $4x + 3y - \lambda = 0$

It will pass through point of intersection (i.e. centre) of $x - y - 1 = 0$ and $3x - 4y - 6 = 0$ which is (–2, –3)

$\therefore \lambda = -17$

$\therefore 4x + 3y + 17 = 0$

11. Pair of angular bisectors : $h[(x-2)^2 - (y+3)^2]$

$2x - 3y = 13$ is one bisector

$\Rightarrow$ other bisector is $3x + 2y = k$ is passing through

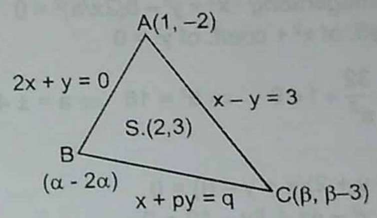
$(2, -3) \Rightarrow 3x + 2y = 0$
$\Rightarrow$ pair of angular bisectors
$(2x - 3y - 13)(3x + 2y) = 0$
$6(x^2 - y^2) - 5xy - 39x - 26y = 0$
compare with
$h(x^2 - y^2) - (a - b)xy - [4h + 3(a - b)]x - [6h + 2(x - b)y] - 5h + 6(a - b) = 0$
$\Rightarrow h = 6, (a - b) = 5$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b} = 1$$

$4(h^2 - ab) = (a + b)^2$
$4[h^2 - ab] = (a - b)^2 + 4ab$

$8ab = 4h^2 - (a - b)^2 = 36 \times 4 - 25 \quad \Rightarrow \quad 8ab = 119 \Rightarrow ab = \frac{119}{8}$

12. solving $2x + y = 0$ and $x - y = 3$, we get $A = (1, -2)$
B lies on $2x + y = 0$
Let $B = (\alpha, -2\alpha)$
C lies on $x - y = 3$
Let $C = (\beta, \beta - 3)$
SA = SB



$(2 - 1)^2 + (3 + 2)^2 = (2 - \alpha)^2 + (3 + 2\alpha)^2$
$5\alpha^2 + 8\alpha - 13 = 0$
$\alpha = 1$ or $\alpha = -13/5$
If $\alpha = 1$, $\beta(1, -2) = A$
$\therefore \alpha \neq 1$
$\alpha = -13/5$

$$B = \left(\frac{-13}{5}, \frac{26}{5}\right)$$

$SC^2 = SA^2$
$(\beta - 2)^2 + (\beta - 6)^2 = (2 - 1)^2 + (3 + 2)^2$
$2\beta^2 - 16\beta + 14 = 0$
$\beta = 1$ or 7
If $\beta = 1$, $c = (1, -2) = A$
$\therefore \beta \neq 1$
$\therefore \beta = 7$ then $c = (7, 4)$

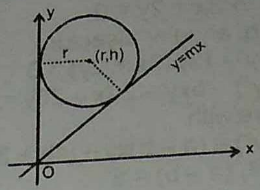
$$\text{Equation of BC} = \frac{y - \frac{26}{5}}{\frac{26}{5} - 4} = \frac{x + \frac{13}{5}}{-\frac{13}{5} - 7}$$

$$\frac{5y - 26}{6} = \frac{5x + 13}{-48}$$

$x + 8y = 39$
comparing with $x + py = q$
$p = 8$, $q = 39$
$pq = 312$

13. $x = 0$ is a tangent
Line through origin is $y - mx = 0$
Applying the condition of tangency $(P = r)$

$\frac{|h-mr|}{\sqrt{1+m^2}} = r \quad h^2 + m^2r^2 - 2mrh = r^2 + r^2m^2$



$m = \frac{h^2-r^2}{2rh}$

$\because$ equation of second tangent

$y = \left(\frac{h^2-r^2}{2rh}\right)x$

$\Rightarrow (h^2 - r^2)x - 2rhy = 0$

14. common chord $x = \frac{a}{2}$

Homogenizing : $x^2 + y^2 - 8(2x/a)^2 = 0$

coeff. of x^2 + coeff. of $y^2 = 0$

$1 - \frac{32}{a^2} + 1 = 0 \quad \Rightarrow a^2 = 16 \quad \Rightarrow a = \pm 4$

15. $(x - y + 2)(x + y - 6) = 0$

$x^2 - y^2 - 4x + 8y - 12 = 0$

$x^2 - y^2 - 4x\left(\frac{\ell x+y}{3}\right) + 8y\left(\frac{\ell x+y}{3}\right) - 12\left(\frac{\ell x+y}{3}\right)^2 = 0$

coefficient of x^2 + coefficient of $y^2 = 0$

$12\ell^2 + 12\ell - 12 = 0$

$\ell^2 + \ell - 1 = 0$

Sum of values of $\ell = -1$

16. Since reflection of the orthocentre of ΔABC in base BC will always lie on the circumcircle of the triangle ABC, therefore coordinate of a point lying on the circumstance is $\left(1-\frac{2.4}{2}, 1-\frac{2.4}{2}\right) = (-3, -3)$ and coordinates of the circumcentre is (2, 0)

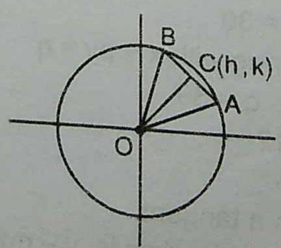
$\therefore$ Radius of the circumcircle of ΔABC is -

$= \sqrt{(3+2)^2+3^2} = \sqrt{34}$

17. since $\angle AOB = 45°$

$\angle COA = 22\frac{1}{2}°$

$\therefore \cos 22\frac{1}{2}° = \frac{\sqrt{h^2+k^2}}{2}$



$\Rightarrow h^2 + k^2 = 4\cos^2 22\frac{1}{2}° = 4\left(\frac{1+\cos 45°}{2}\right) = 2\left(1+\frac{1}{\sqrt{2}}\right)$

$= 2 + \sqrt{2}$

18. Here $e = \frac{3}{5}$ and given $\frac{ar.\Delta PTN}{ar\Delta PSS'} = \frac{91}{90}$

$\Rightarrow \quad \frac{a\sec\theta - ae^2\cos\theta}{2ae} = \frac{91}{90}$

$\Rightarrow \quad 50\sec\theta - 18\cos\theta = 91$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \qquad \therefore \quad \sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \Delta PSS' = \frac{1}{2} \times 6 \times 4 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$ sq. units

19. The tangents and normal form a rectangle
hence tangents meet on the directrix.
$(y-2)^2 = 2(x+2)$ vertex $(-2, 2)$

Directrix $x = -\frac{5}{2}$

20. $4\tan\frac{B}{2} \cdot \tan\frac{C}{2} = 1$

$\Rightarrow \sqrt{\frac{(s-c)(s-a)}{s(s-b)} \frac{(s-a)(s-b)}{s(s-c)}} = \frac{1}{4}$

$\Rightarrow \frac{s-a}{s} = \frac{1}{4} \Rightarrow \frac{2s-a}{a} = \frac{5}{3}$

$\Rightarrow b + c = \frac{5}{3} \times 6 = 10 \quad (\because a = BC = 6)$

21. The equation represents the parallel lines $x - y + 1 = 0$, $2x - 2y - 1 = 0$. The distance between them is $\frac{3}{2\sqrt{2}}$

The distance between the parallel lines is $2\sqrt{\frac{g^2 - ac}{a(a+b)}}$

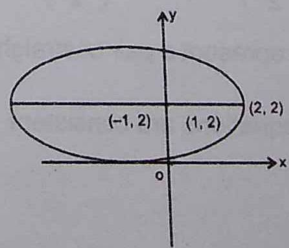
22. $xy - 3x - 4y + 8 = 0 \Rightarrow (x-4)(y-3) = 4$
which is a rectangular hyperbola of the type $xy = c^2$
$\therefore c = 2$

$\therefore a = b = c\sqrt{2} = 2\sqrt{2}$

$\therefore$ length of latus rectum $= \frac{2b^2}{a} = 2a \ (\because a = b) = 4\sqrt{2}$

23. $4x^2 + 8x + 9y^2 - 36y = -4$

$4(x^2 + 2x + 1) + 9(y^2 - 4y + 4) = 36$



$$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$$

$\therefore$ G = 5, L = 1

24. $ax^2 + bx + c = 0$

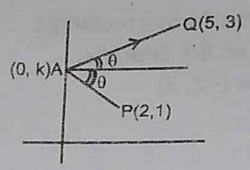
Since real parts are negative

$\therefore$ sum of roots is negative i.e. $-\frac{b}{a} < 0$ i.e. $ab > 0$ ....(i)

product of roots $= \frac{c}{a} > 0$ i.e. $ac > 0$ ....(ii)

from (i) and (ii) a, b, c are of the same sign and $a \neq 0, b \neq 0, c \neq 0$

25. Slope of AQ = – Slope of AP



i.e. $\frac{k-3}{-5} = -\frac{k-1}{-2}$

$\Rightarrow k = \frac{11}{7}$

26. $y = x^2 - ax - 1$

$\therefore x = \frac{a \pm \sqrt{a^2+4}}{2}$

$\alpha = \frac{a+\sqrt{a^2+4}}{2}, \beta = \frac{a-\sqrt{a^2+4}}{2}$

Equation of family of circles through A and B is

$(x-\alpha)(x-\beta) + y^2 + \lambda y = 0$

As it passes through C(0, –1)

$\alpha\beta + 1 - \lambda = 0$ (But $\alpha\beta = -1$)

$\therefore \lambda = 0$

Equation of circle through A, B and C is

$(x-\alpha)(x-\beta) + y^2 = 0$

It cuts the y-axis when x = 0, so $\alpha\beta + y^2 = 0$ (put $\alpha\beta = -1$)

$\therefore y^2 = 1 \Rightarrow y = 1$ or -1

Hence t = 1 **Ans.**

27. Given $12x^2 - 7xy - 12y^2 - 5x + 90y - 150 = 0$

Here $abc + 2fgh - af^2 - bg^2 - ch^2 = 12 \cdot (-12)(-150) + 2.45 \cdot \left(\frac{-5}{2}\right)\left(\frac{-7}{2}\right) - 12 \cdot (45)^2$

$-(-12)\left(\frac{-5}{2}\right)^2 - (-150)\left(\frac{-7}{2}\right)^2 = 0$

$\therefore$ It represent a pair of straight lines

28. Since the equations are consistent

$\therefore D = 0$

$$\begin{vmatrix} (a+1)^3 & (a+2)^3 & -(a+3)^3 \\ (a+1) & (a+2) & -(a+3) \\ 1 & 1 & -1 \end{vmatrix} = 0$$

Put u = (a + 1), v = a + 2, w = a + 3
u – v = – 1, v – w = – 1, w – u = 2
u + v + w = 3a + 6

$$\therefore \begin{vmatrix} u^3 & v^3 & w^3 \\ u & v & w \\ 1 & 1 & 1 \end{vmatrix} = 0$$

(u – v) (v – w) (w – u) (u + v + w) = 0
(–1) (–1) (2) (3a + 6) = 0
i.e. a = – 2

29. Focus (2, 3)
Directrix 3x + 4y – 1 = 0

$$L/R = 2\left|\frac{6+12-1}{5}\right| = \frac{34}{5}.$$

30. $ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2e^2$
$\ell x + my = 1$

$$\frac{a\sec\theta}{\ell} = \frac{-b\,\mathrm{cosec}\theta}{m} = a^2e^2$$

$$\cos\theta = \frac{1}{ae^2\ell},\ \sin\theta = -\frac{b}{a^2e^2m}$$

$$\frac{1}{a^2e^4\ell^2} + \frac{b^2}{a^4e^4m^2} = 1$$

SOLUTION TO PART TEST-2 TOPIC : ALGEBRA - 1 (XI)

Answers

1.	(3)	2.	(2)	3.	(4)	4.	(2)	5.	(4)	6.	(2)	7.	(2)
8.	(2)	9.	(2)	10.	(3)	11.	(1)	12.	(2)	13.	(1)	14.	(4)
15.	(3)	16.	(1)	17.	(3)	18.	(1)	19.	(1)	20.	(4)	21.	(4)
22.	(1)	23.	(3)	24.	(1)	25.	(4)	26.	(2)	27.	(2)	28.	(2)
29.	(4)	30.	(4)										

1. a, b, c are in AP $\Rightarrow c + a = 2b$ .... (i)

a^2, b^2, c^2 are in HP $\Rightarrow b^2 = \dfrac{2a^2c^2}{a^2+c^2}$

$\Rightarrow b^2(a^2 + c^2) - 2a^2c^2 = 0$

$b^2[(c + a)^2 - 2ca) - 2a^2c^2 = 0$

$\Rightarrow b^2[4b^2 - 2ca) - 2a^2c^2 = 0$ by using (i)

$(b^2 - ca)(2b^2 + ca) = 0 \quad \therefore\ b^2 = ca\ ,\ -\dfrac{ca}{2}$

2. Let root of $ax^2 + bx + c = 0$ be α , β

Roots of $2x^2 + 8x + 2 = 0$ are $\alpha - 1$, $\beta - 1$

Let $y = x + 1$ $\therefore$ the second equation becomes

$2(y - 1)^2 + 8(y - 1) + 2 = 0$

$\Rightarrow y^2 + 2y - 2 = 0$

The roots of this equation are

$(\alpha - 1) + 1 = \alpha$ and $(\beta - 1) + 1 = \beta$

This equation in variable x is $x^2 + 2x - 2 = 0$

This equation is same as $ax^2 + bx + c = 0$

$\therefore \dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{-2} \Rightarrow b = 2a\ ,\ b = -c\ ,\ c = -2a$

$\Rightarrow b + c = 0$

also $c + a = \dfrac{b}{2} + (-b) = -\dfrac{b}{2}$

3. $\alpha + \beta = -p\ ,\ \alpha\beta = q$

$\alpha^2 + \beta^2 = p^2 - 2q$

$\alpha^4 + \alpha^2\beta^2 + \beta^4 = (\alpha^2 + \beta^2)^2 - \alpha^2\beta^2 = (p^2 - 2q)^2 - q^2 =$

$= (p^2 - q)(p^2 - 3q)$

4. $|x^2 - x + 1| = |x^2 - 2x + 3|$

$\Rightarrow \left|\left(x - \dfrac{1}{2}\right)^2 + \dfrac{3}{4}\right| = |(x - 1)^2 + 2|$

$\left(x - \dfrac{1}{2}\right)^2 + \dfrac{3}{4} = (x - 1)^2 + 2$

$\Rightarrow x = 2$

5. $\dfrac{m+n}{2} = \sqrt{ab} = \dfrac{ma+nb}{m+n}$

$(m + n)\sqrt{ab} = ma + nb$

$m(\sqrt{b} - \sqrt{a})\sqrt{a} = n(\sqrt{b} - \sqrt{a})\sqrt{b}$

$\frac{m}{\sqrt{b}} = \frac{n}{\sqrt{a}} = k$, say .... (i)

further $\frac{m+n}{2} = \sqrt{ab}$ ...... (ii)

from (i) , (ii) $\Rightarrow \frac{k}{2}(\sqrt{b}+\sqrt{a}) = \sqrt{ab}$ $\quad \therefore\ k = \frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

$\Rightarrow\ m = k\sqrt{b} = \frac{2b\sqrt{a}}{\sqrt{a}+\sqrt{b}}$

6. let the GP be a, ar, ar^2, ...

$\frac{a}{1-r} = 3$ ...... (i)

The sequence of squares of terms is a^2, a^2r^2, a^2r^4, ...

sum = $\frac{a^2}{1-r^2} = \frac{9}{2}$ .... (ii)

from (i) , (ii) $(3(1-r))^2 = \frac{9(1-r^2)}{2} \Rightarrow r = \frac{1}{3}$

$\therefore\ a = 3(1-r) = 3\left(1-\frac{1}{3}\right) = 2$

The sequence of cubes of terms is a^3, a^3r^3, a^3r^6, ...

sum = $\frac{a^3}{1-r^3} = \frac{2^3}{1-\left(\frac{1}{3}\right)^3} = \frac{108}{13}$

7. $b > 0$ and $bx^2 + \sqrt{(a+c)^2+4b^2}\,x + (a+c) \geq 0\ \forall\ x \in R$

$\Rightarrow D \leq 0$

$\Rightarrow (a+c)^2 + 4b^2 - 4b(a+c) \leq 0$

$\Rightarrow (a+c-2b)^2 \leq 0 \Rightarrow a+c-2b = 0$

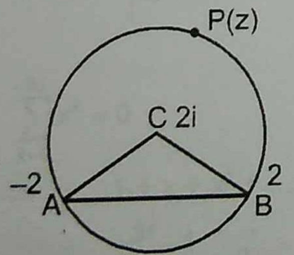
$\Rightarrow$ a, b, c are in AP

8. $|z^2-3| \geq |z|^2 - 3 \Rightarrow 3|z| \geq |z|^2 - 3$

$\Rightarrow |z|^2 - 3|z| - 3 \leq 0$

$0 < |z| \leq \frac{3+\sqrt{21}}{2}$

9. $|z-2i| = 2\sqrt{2}$ is a circle



with centre 2i which passes through the points – 2 and 2.

AB subtends $\frac{\pi}{2}$ at C and $\pi/4$ at z.

10. $z = x + iy$, $x < 0$, $y < 0$

$$\therefore \frac{\bar{z}}{z} = \frac{x - iy}{x + iy} = \frac{x^2 - y^2}{x^2 + y^2} - \left(\frac{2xy}{x^2 + y^2}\right)i$$

$\frac{\bar{z}}{z}$ lies in III quadrant if $x^2 - y^2 < 0$ and $-2xy < 0$

$x < 0$, $y < 0$ $\Rightarrow$ $-2xy < 0$
also $x^2 - y^2 < 0$ $\Rightarrow$ $(x + y)(x - y) < 0$
If $x - y > 0$ $\Rightarrow$ $x > y$ ($\because$ $x, y < 0 \Rightarrow x + y < 0$)

$\frac{\bar{z}}{z}$ lies in III quadrant if $y < x < 0$

11. $z = re^{i\pi/4} \Rightarrow z^2 = r^2 . e^{i\pi/2} = ir^2$
Re $z^2 = 0$

12. $x^2 - (m + 1)x + m + 4 = 0$
since roots are real, we have
$(-(m + 1))^2 - 4.1.(m+4) \geq 0$
$\Rightarrow m^2 - 2m - 15 \geq 0 \Rightarrow (m + 3)(m - 5) \geq 0$
$m \leq -3$ or $m \geq 5$ .... (1)
Since both roots negative then their product must be positive & sum must be negative

i.e. $\alpha+\beta < 0 \Rightarrow m+1 < 0 \Rightarrow m < -1$ and $\frac{m+4}{1} > 0$ or $m > -4$ .... (2)

combing (1) and (2) $-4 < m \leq -3$

13. $x^{2n} - 1 = 0$ $(n \in N)$
$x^{2n} - 1 = 0 \Rightarrow x^{2n} = 1 = \cos 0 + i\sin 0$

$x = \cos\frac{2r\pi}{2n} + i\sin\frac{2r\pi}{2n}$

$= \cos\frac{r\pi}{n} + i\sin\frac{r\pi}{n}$, $r = 0, 1, ... (2n - 1)$

x will be real only when $\sin\frac{r\pi}{n} = 0$

or $\frac{r\pi}{n} = m\pi$ or $r = mn$ = a multiple of n

But $r = 0, 1, ... 2n - 1$
$r = 0, n$
Equation $x^{24} - 1 = 0$ has only real roots $1, -1$

14. It is not an AP or GP or HP
It is an AGP
$3, (3 + d)r, (3 + 2d)r^2, (3 + 3d)r^3, ...$
$(3 + d)r = -1$, $(3 + 2d)r^2 = -1$
Eliminate r , $(3 + d)^2 = -(3 + 2d)$
$d^2 + 8d + 12 = 0$ $\therefore d = -2$, $r = -1$
$d = -6$, $r = 1/3$
The next term $= (3 + 3d)r^3 = 3, -5/9$

15. The centres of circles are $-a$ and $-c$ and radii $\sqrt{a\bar{a} - b}$ and $\sqrt{c\bar{c} - d}$
The circles cut orthogonally
$\therefore |c - a|^2 = (c - a)(\bar{c} - \bar{a}) = a\bar{a} - b + c\bar{c} - d$
$\Rightarrow c\bar{a} + \bar{c}a = b + d$

$\text{Re}(a\bar{c}) = \frac{1}{2}(b + d)$

16. $\frac{1}{\sqrt{n}(n+1)} = \frac{\sqrt{n}}{n(n+1)} = \frac{\sqrt{n}}{n} - \frac{\sqrt{n}}{n+1}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n+1)\sqrt{n}} = 1 + \sum_{n=2}^{\infty} \frac{\sqrt{n} - \sqrt{n-1}}{n} \le 1 + \sum_{n=2}^{\infty} \frac{\sqrt{n} - \sqrt{n-1}}{\sqrt{n}\sqrt{n-1}}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n+1)\sqrt{n}} \le 1 + \sum_{n=2}^{\infty} \left(\frac{1}{\sqrt{n-1}} - \frac{1}{\sqrt{n}}\right) < 2$

17. $\because$ sum of the cofficient of equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ in zero.

$\therefore$ roots are 1 & $\frac{c(a-b)}{a(b-c)}$

If 1 is also root of $px^2 + qx + r = 0$ then $p + q + r = 0$

$\Rightarrow p^3 + q^3 + r^3 = 3pqr$

18. Given G.P. $a(y^2 - 4y + 5) + a + \frac{a}{y^2 - 4y + 5} + \frac{a}{(y^2 - 4y + 5)^2} + \ldots$

Sum of G.P. is finite if common ratio $|r| < 1$

$|(y-2)^2 + 1| > 1 \Rightarrow y \ne 2$

$\Rightarrow x^2 - 6x + 11 \ne 2$

$\Rightarrow x \ne 3$

19. If equation has exactly one root between 1 and 2 than -

$f(1) . f(2) < 0 \Rightarrow (e^\theta - e^{2\theta} + e^\theta - 1)\ (4e^\theta - 2e^{2\theta} + e^\theta - 1) < 0$

$(2e^\theta - e^{2\theta} - 1)\ (5e^\theta - 2e^{2\theta} - 1) < 0$

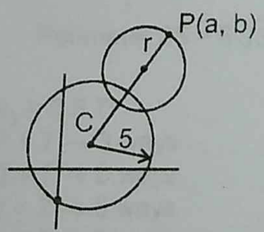
$[e^\theta - 1]^2\ [2e^{2\theta} - 5e^\theta + 1] < 0$

$\Rightarrow \theta \in \left(\log\frac{5-\sqrt{17}}{4}, \log\frac{5+\sqrt{17}}{4}\right)$

20. Radius of smallest circle r = CP – 5

Radius of largest circle R = CP + 5

hence difference = 10



21. $\sum_{k=0}^{30} x^k = 0$

$\Rightarrow \quad 1 + x + x^2 + \ldots\ldots\ldots + x^{30} = 0$

$\Rightarrow \quad \frac{x^{31} - 1}{x - 1} = 0$

$\frac{1}{\alpha - 1} = x \quad \Rightarrow \quad \alpha = 1 + \frac{1}{x}$

$$\frac{\left(1+\frac{1}{x}\right)^{31}-1}{1/x}=0$$

$\Rightarrow \quad (x+1)^{31}-x^{31}=0$

$\Rightarrow \quad {}^{31}C_0x^{31}+{}^{31}C_1x^{30}+{}^{31}C_2x^{29}+......+{}^{31}C_{31}-x^{31}=0$

$\Rightarrow \quad {}^{31}C_1x^{30}+{}^{31}C_2x^{29}+......+1=0$ ...(1)

Now, $\sum_{i=1}^{30}\frac{1}{\alpha_i-1}$ = Sum of roots of equation (1)

$=\frac{{}^{31}C_2}{31}=-15$

22. Let α and β be the roots of $x^2-ax+b=0$

$\alpha+\beta=a;\ \alpha\beta=b$

$|\alpha-\beta|<c\Rightarrow(\alpha-\beta)^2<c^2$

$(\alpha+\beta)^2-4\alpha\beta<c^2$

$a^2-c^2<4b$

$\frac{1}{4}(a^2-c^2)<b$ ...(1)

The roots are real

$\Delta\geq 0$

$a^2-4b\geq 0\Rightarrow a^2>4b$

$b\leq\frac{a^2}{4}$ ...(2)

From (1) & (2)

$\frac{1}{4}(a^2-c^2)<b\leq\frac{a^2}{4}$

23. Let the numbers be $\alpha,\ \alpha r,\ \alpha r^2$

$\alpha(1+r+r^2)=ka$

$\alpha^2(1+r^2+r^4)=k^2b\Rightarrow\frac{k^2b}{k^2a^2}=\frac{1+r^2+r^4}{(1+r+r^2)^2}$

$\Rightarrow\frac{b}{a^2}=\frac{(1+r^2)^2-r^2}{(1+r+r^2)^2}=\frac{1+r^2-r}{1+r+r^2}$

$\Rightarrow(a^2-b)r^2-(a^2+b)r+(a^2-b)=0$

r is real $\Rightarrow\Delta\geq 0$ but $\Delta=0$ we will get $r=1$ which is not possible

$\Rightarrow(a^2+b)^2-4(a^2-b)^2>0$ and $a^2-b>0$

$\Rightarrow\frac{a^2+b}{a^2-b}>2$ and $a^2>b$

$\Rightarrow a^2+b>2(a^2-b)$ and $a^2>b\Rightarrow a^2<3b,\ a^2>b\Rightarrow 1<\frac{a^2}{b}<3$

24. $b+c$, $c+a$, $a+b$ are in H.P.

$(c+a)=\frac{2(b+c)(a+b)}{2b+a+c}$

$\Rightarrow\quad a^2+c^2=2b^2$

Now If a, b, c are in G.P. $\Rightarrow\ b^2=ac$

$\Rightarrow\ a^2+c^2=2ac\ \Rightarrow\ (a-c)^2=0\ \Rightarrow\ a=b=c$

Hence ΔABC is equilatral

25. Given expression = $x^2\{{}^nC_1 - {}^nC_2 + ... + (-1)^{n-1}\,{}^nC_n\} - 2x\{{}^nC_1 - 2\,{}^nC_2 + ... + (-1)^{n-1}\,n\,{}^nC_n\}$
$+ \{1^2\,{}^nC_1 - 2^2\,{}^nC_2 + ... (-1)^{n-1}\,{}^nC_n n^2\} = x^2$

26. $(3-2x)^{-1/2} = 3^{-1/2}\left(1-\frac{2x}{3}\right)^{-\frac{1}{2}}$

For expansion to be valid $0 < \left|\frac{2x}{3}\right| < 1 \Rightarrow \quad x \in \left(-\frac{3}{2}, \frac{3}{2}\right), x \neq 0$

$\Rightarrow \quad a = -\frac{3}{2}\ ;\ b = \frac{3}{2}$

$\therefore \quad 4a + 6b = 3$

27. $0 + 1 + 3 + 5 + 7 = 16$

For a number to be divisible by 6, it has to be divisible by both 2 and 3. Hence '0' has to be included for divisibility by 2.

Hence digits can be (0, 3, 5, 7) or (0, 1, 3, 5)

$\Rightarrow$ | 3 | 5 | 7 | 0 | = 3 !

| 1 | 3 | 5 | 0 | = 3 !

28. 2^p when divided by 4 the remainder is 0 or 2

3^q when divided by 4 the remainder is 3 or 1

5^r when divided by 4 the remainder is 1

$\therefore \quad 2^p + 3^q + 5^r$ is divisible by 4

if the combinations for the remainder are 0, 3, 1; 2, 1, 1

number of ways for the first set = 5 × 5 × 10 = 250

number of ways for the second set = 5 × 5 × 10 = 250

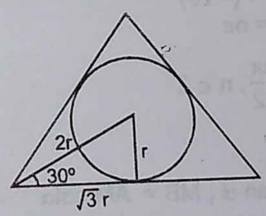
$\therefore$ total number of ways = 500

29. r = 4R sin A/2 sin B/2 sin C/2

(1) R = 2r rational

(2) $r_1 = \frac{\Delta}{S-a} = \frac{3\sqrt{3}r^2}{3\sqrt{3}r - 2\sqrt{3}r} = 3r$ rational

(3) $\Delta = \frac{\sqrt{3}}{4}(2\sqrt{3}\ r)^2 = 3\sqrt{3}\ r^2$ irrational



(4) Perimeter $2S = 6\sqrt{3}\ r$ which is not rational

30. $x_1 x_2 x_3 = 2.5.7^2$

$10 \times 7 \times 7 \rightarrow 3$ ways

$14 \times 5 \times 7 \rightarrow 6$ ways

$35 \times 2 \times 7 \rightarrow 6$ ways

$2 \times 5 \times 49 \rightarrow$ <u>6 ways</u>

Total number of ways = 21

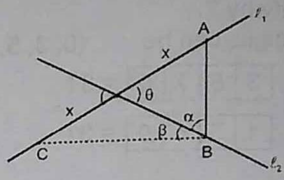
SOLUTION TO PART TEST-3 TOPIC : ALGEBRA- 2 & TRIGONOMETRY (XI)

Answers

1.	(2)	2.	(1)	3.	(2)	4.	(3)	5.	(3)	6.	(1)	7.	(4)
8.	(1)	9.	(3)	10.	(3)	11.	(3)	12.	(2)	13.	(2)	14.	(3)
15.	(1)	16.	(2)	17.	(2)	18.	(3)	19.	(3)	20.	(4)	21.	(2)
22.	(2)	23.	(1)	24.	(4)	25.	(2)	26.	(2)	27.	(4)	28.	(4)
29.	(1)	30.	(4)										

SOLUTIONS

1. ℓ_1 and ℓ_2 are two intersecting railway lines
In ΔABC,
by m – n theorem, we have $(x + x)\cot\theta = x\cot\beta - x\cot\alpha$
$\Rightarrow 2\cot\theta = \cot\beta - \cot\alpha$



2. $\tan\theta = \tan\left(\frac{\pi}{2} - \alpha\right)$

$\therefore \theta = n\pi + \frac{\pi}{2} - \alpha$

3. Given equation can be written as
$\tan\theta + \tan4\theta = -\tan7\theta\,(1 - \tan\theta\tan4\theta)$
$\Rightarrow -\tan7\theta = \tan(\theta + 4\theta)$
$\Rightarrow \tan5\theta = \tan(-7\theta)$
$5\theta = n\pi + (-7\theta)$
$\therefore 12\theta = n\pi$

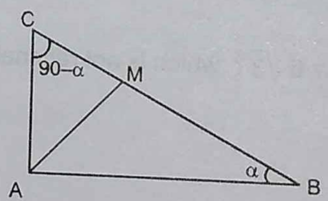
$\therefore \theta = \frac{n\pi}{12}, n \in Z$

4. $\frac{AM}{MB} = \tan\alpha$; $MB = AM\cot\alpha$

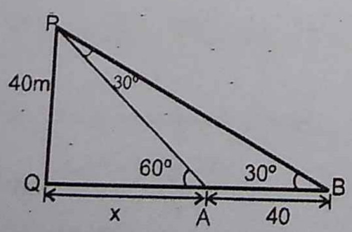
$\frac{AM}{MC} = \cot\alpha \Rightarrow MC = AM\tan\alpha$

$(MB + MC) = AM(\cot\alpha + \tan\alpha)$

$\Rightarrow \frac{1}{\sin2\alpha} = 2 = \alpha = 15^\circ$



5.



Let breadth of the river be x m

$\therefore \frac{PQ}{40 + x} = \tan30^\circ$ and $\frac{PQ}{x} = \tan60^\circ$

$\therefore$ x = 20 m

6. If we choose k $(0 \le k \le n)$ identical objects, then we must choose (n –k) distinct object. This can be done in $^{2n+1}C_{n-k}$ ways. Thus, the required number of ways.

$$= \sum_{k=0}^{n} {}^{2n+1}C_{n-k} = {}^{2n+1}C_n + {}^{2n+1}C_{n-1} + \text{----} + {}^{2n+1}C_0 = 2^{2n}.$$

7. The required number of ways
the number of ways in which 8 girls can sit – the number of ways in which two sisters are together

$= \lfloor 8 - 2 \lfloor 7 = 30240$

8. We first arrange the m men . This can be done is $\lfloor m$ ways. After m men have taken their seats, the women must chose w seats out of (m + 1) seats marked with X below.

$\underset{\text{I}}{\text{XM}}\underset{\text{II}}{\text{XM}}\underset{\text{III}}{\text{XM}}\text{X.....}\underset{m+n}{\text{XM}}\text{X}$

They can choose w seats in $^{m+1}C_w$ ways and take their seats in $\lfloor w$ ways.

Thus, the required numbers of arrangements is

$$\lfloor m \left({}^{m+1}C_w\right)\lfloor w = \frac{\lfloor m \lfloor m+1}{\lfloor m+1-w}$$

9. Multiplying nonvector and Denominator of E by 1– x we have

$$E = \frac{(1-x)}{(1-x)(1+x)(1+x^2)----(1+x^{2^m})}$$

$$= (1-x)\ (1-x^{2^{m+1}})^{-1}$$

$$= (1-x)\ (1+x^{2^{m+1}} + x^{2^{m+2}} + ...)$$

coefficient of $x^{2^{m+1}}$ is 1.

10. $T_{r+1} = {}^nC_r\,(-1)^r\,x^{3n-5r}$

For x^5 ; $3n - 5r = 5$

$$\therefore r = \frac{3n-5}{5} = P\ (\text{say})$$

and for x^{10} ; $3n - 5r = 10$

$$\therefore r = \frac{3n-10}{5} = q\ (\text{say})$$

Note p – q = 1 we have

$^nC_p\,(-1)^p + {}^nC_q\,(-1)^q = 0$

$\Rightarrow {}^nC_p\,(-1)^p + {}^nC_{p-1}\,(-1)^{p-1} = 0$

$^nC_p = {}^nC_{p-1}$

$\Rightarrow n - p = p - 1$

$\therefore p = \left(\frac{n+1}{2}\right)$

$\therefore \frac{n+1}{2} = \frac{3n-5}{5}$

$\therefore n = 15$

11. putting $x = i$

$\Rightarrow 1 = (a_0 - a_2 + a_4 - ....) + i\,(a_1 - a_3 + ...)$

equating the real part we get

$a_0 - a_2 + a_4 - a_6 + ---- + a_{96} = 1$

12. sum of given series

$$= \sum_{r=0}^{n} (4r+1)C_r$$

$$= 4\sum_{r=0}^{n} rC_r + \sum_{r=0}^{n} C_r$$

$$= 4n\sum_{r=1}^{n} {}^{n-1}C_{r-1} + 2^n$$

$= 4n\, 2^{n-1} + 2^n$

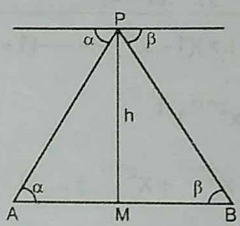
$= 2^n (1 + 2n)$

13. Let MP be the light house and A,B be the two stations, then

$a = AB = AM + MB$

$\Rightarrow \frac{a}{h} = \frac{AM}{h} + \frac{MB}{h}$

$\Rightarrow \frac{a}{h} = \cot\alpha + \cot\beta \Rightarrow h = \frac{a}{\cot\alpha + \cot\beta}$



14. $\tan 3\theta \tan 2\theta = 1$

$\Rightarrow \cos 3\theta \cos 2\theta - \sin 3\theta \sin 2\theta = 0$

$\Rightarrow \cos 5\theta = 0,\ \cos 3\theta \neq 0,\ \cos 2\theta \neq 0$

$2\theta, 3\theta, \neq$ an odd multiple of $\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\frac{\pi}{10}$ where $2n + 1 \neq 5k,\ k \in I$

($\because$ If $2n + 1$ is divided by 5, then $(2n + 1)\frac{\pi}{10}$ is an odd multiple of $\frac{\pi}{2}$ for which $\tan 3\theta$ is n to defined.)

15. $x > y \Rightarrow \frac{x}{\sqrt{1-x^2}} > y \Rightarrow \tan^{-1}\frac{x}{\sqrt{1-x^2}} > \tan^{-1} y$

$\Rightarrow \sin^{-1} x > \tan^{-1} y$

$e < \pi \Rightarrow \frac{1}{\sqrt{e}} > \frac{1}{\sqrt{\pi}} \Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$

16. $2^p \cdot 6^q \cdot 15^r = 2^{p+q} \cdot 3^{q+r} \cdot 5^r$

Proper divisors $\equiv (p + q + 1)(q + r + 1)(r + 1) - 2$

17. ${}^nC_4\, a^{n-4} b^4 - {}^nC_5\, a^{n-5} b^5 = 0$

${}^nC_4\, a - {}^nC_5 - b = 0$

$$\frac{a}{b} = \frac{{}^nC_5}{{}^nC_4} = \frac{\lfloor n \times \lfloor n-4}{\lfloor n-5\ \lfloor 5 \times \lfloor n} \lfloor 4 = \frac{n-4}{5}$$

18. $T_{r+1} = 600_{C_r} \quad (7^{1/3})^{600-r} \quad (5^{1/2})^r$

↓ r is multiple of 3 ↓ r is multiple of 2

$\Rightarrow$ multiple of $6 = 0, 6, 12.........600$

101 values

19. put n = 1, 2, 3........ It is true for all $n \in N$

20. $A = \pi - (B + C)$

$\tan A = \dfrac{\tan B + \tan C}{\tan B \tan C - 1}$ –(Statement 2)

A is obtuse

$\tan A < 0$

$\Rightarrow \tan B \tan C - 1 < 0$ [B & C acute]

$\tan B \tan C < 1$

21. $$\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{s^2 + (s-a)^2 + (s-b)^2 + (s-c)^2}{\Delta^2}$$

$$= \frac{4s^2 - 2s(a+b+c) + a^2 + b^2 + c^2}{\Delta^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$$

$\therefore$ n = 2

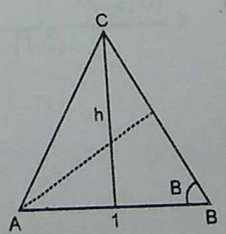
22. Number of terms in binomial expension ${}^{n+k-1}C_{k-1}$

23. Statement 1 :- $\dfrac{a^2 + b^2 + c^2}{R^2} = \dfrac{2a^2}{R^2} = 8\left[\dfrac{a}{2R}\right]^2 = 8[\sin 90^\circ]^2$

$= 8$ [A = 90°]

24. product of altitudes P = h sin B sin A

Also h cot A + h cot B = 1



$\Rightarrow$ h sin (A + B) = sin A sin B

$\Rightarrow$ $P = h^2 \sin(A + B)$ this is maximum when $A + B = \dfrac{\pi}{2}$ i.e. when Δ is right angled.

25. $\sec^2\theta_n = \sec^2\theta_{n-1} + 4 + 4\cos^2\theta_{n-1}$

$\because \quad 0 < 4\cos^2\theta_{n-1} < 4$

$4 + \sec^2\theta_{n-1} < \sec^2\theta_n < 8 + \sec^2\theta_{n-1}$

$4n < \sec^2\theta_n < 8n - 4$

$2\sqrt{n} < |\sec\theta_n| < \sqrt{8n-4}$

$n = 4 \quad 4 < |\sec\theta_4| < 2\sqrt{7} < 6$

$n = 5 \quad 2\sqrt{5} < |\sec\theta_5| < 6$

26. $\dfrac{\cos^2\frac{A}{2}}{a} = \dfrac{s(s-a)}{abc}$

$\therefore \quad \sum\dfrac{\cos^2\frac{A}{2}}{a} = \dfrac{s^2}{abc}$

27. Let $y = \dfrac{\tan\theta}{\tan 3\theta} = \dfrac{1-3\tan^2\theta}{3-\tan^2\theta}$

$\Rightarrow \tan^2\theta = \dfrac{3y-1}{y-3} > 0 \Rightarrow y \in \left(-\infty, \dfrac{1}{3}\right) \cup (3, \infty)$

28. $\cos^{-1}\left(\dfrac{x}{2}\right) + \cos^{-1}\left(\dfrac{y}{3}\right) = \theta$

$\therefore \quad \cos\theta = \dfrac{x}{2}\cdot\dfrac{y}{3} - \sqrt{1-\dfrac{x^2}{4}}\sqrt{1-\dfrac{y^2}{9}}$

i.e. $\quad xy - 6\cos\theta = \sqrt{4-x^2}\sqrt{9-y^2}$

$\therefore \quad 9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta \le 36$

29. Given expression $= -\cos\dfrac{3\pi}{17}\cos\dfrac{6\pi}{17}\cos\dfrac{12\pi}{17}\cos\dfrac{24\pi}{17}$

$= \dfrac{-1}{2^4}\dfrac{\sin\left(\frac{48\pi}{17}\right)}{\sin\left(\frac{3\pi}{17}\right)} = -\dfrac{1}{16}\dfrac{\sin\left(3\pi - \frac{3\pi}{17}\right)}{\sin\frac{3\pi}{17}} = \dfrac{-1}{16}$

30. Though $P(n) \Rightarrow P(n+1)$ for all $n \in N$ yet nothing can be said about truth of $P(n)$ in general as $P(1)$ is not given to be true.

SOLUTION TO PART TEST-4 TOPIC : DIFFERENTIAL CALCULUS (XII)

Answers

1.	(2)	2.	(1)	3.	(4)	4.	(4)	5.	(1)	6.	(2)	7.	(1)
8.	(4)	9.	(3)	10.	(4)	11.	(1)	12.	(2)	13.	(4)	14.	(3)
15.	(2)	16.	(1)	17.	(2)	18.	(4)	19.	(3)	20.	(3)	21.	(4)
22.	(2)	23.	(1)	24.	(1)	25.	(2)	26.	(1)	27.	(1)	28.	(4)
29.	(2)	30.	(1)										

SOLUTIONS

1. $f(x) = \cos 8\pi \{x\} = \cos (8\pi x - 8\pi [x]) = \cos 8\pi x$. Its period is $\frac{1}{4}$.

Period of $\sin 2\pi x \operatorname{cosec} 2\pi x$ is $\frac{1}{2}$

$\therefore$ period of f(x) is $\frac{1}{2}$

2. $\frac{1}{|x|-1}$ is discontinuous at $x = -1, 1$

and tanx is continuous at $x = -1, 1$

$\Rightarrow$ f(x) is discontinuous at $x = -1, 1$

Also tanx is discontinuous at $x = -\frac{\pi}{2}, \frac{\pi}{2}$

and $\frac{1}{|x|-1}$ is continuous at $x = -\frac{\pi}{2}, \frac{\pi}{2}$

$\Rightarrow$ f(x) is discontinuous at $x = -\frac{\pi}{2}, \frac{\pi}{2}$

Hence f (x) is discontinuous at four points.

3. $(x^2 - 3x - 10)(\log^2 (x-3)) \geq 0$

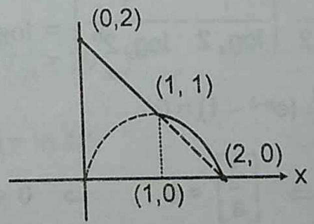
$x - 3 > 0$ & $(x^2 - 3x - 10) \geq 0$ or $x = 4$

$\Rightarrow$ $x > 3$ ..... (i) $x \in (-\infty, -2] \cup [5, \infty) \cup \{4\}$ ..... (ii)

$x = 4$ also element of domain ..... (iii)

Common part of (i), (ii) & (iii) $x \in [5, \infty) \cup \{4\}$

4. At x = 2



$$\text{L.H.D} = \lim_{h \to 0^+} \frac{\sqrt{(2-h)(2-(2-h))} - 0}{-h}$$

$$= \lim_{h \to 0^+} \frac{\sqrt{2-h}}{-\sqrt{h}} \left(\frac{\sqrt{2}}{0} \text{form}\right)$$

= not finite.

5. $x^5 + 2x^3 + 2x = -5$

$x^5 + 2x^3 + 2x + 5 = 0$

we get $x = -1$

Since $fog(x) = x$ $\Rightarrow$ $f(g(x)) = x$

$$f'(g(x))\, g'(x) = 1 \qquad \Rightarrow g'(x) = \frac{1}{f'(g(x))}$$

$$g'(-5) = \frac{1}{f'(-1)} = \frac{1}{5(-1)^4 + 6(-1)^2 + 2} = \frac{1}{13}$$

6. $2\tan^{-1} p - 2\tan^{-1} q = 2\tan^{-1} x$

$$\tan^{-1}\frac{p-q}{1+pq} = \tan^{-1}x$$

$$x = \frac{p-q}{1+pq}$$

7. y = x and y = sin x intersect at only one point (0, 0)
$\Rightarrow$ $A \cap B$ is singleton set

8. $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 12 + 9 - 4 = 17$
Now, $n((A \cup B)^c) = n(U) - n(A \cup B) = 20 - 17 = 3$.

9. Obviously (3)

10. Let f(x) be periodic with period λ, $\lambda > 0$
$\therefore f(x + \lambda) = f(x)$
$\Rightarrow \cos(\cos(x + \lambda)) + \cos(\sin(x + \lambda)) + \sin(4(x + \lambda))$
$= \cos(\cos x) + \cos(\sin x) + \sin 4x$
put x = 0
$\cos(\cos\lambda) + \cos(\sin\lambda) + \sin 4\lambda = \cos 1 + \cos 0 + \sin 0$

$$= \cos\left(\sin\frac{\pi}{2}\right) + \cos\left(\cos\frac{\pi}{2}\right) + \sin 2\pi \qquad \Rightarrow \lambda = \frac{\pi}{2}$$

11. $0 \le \{x\} < 1$
$0 \le e\{x\} < e$
$\ell n\, 0 < \ell n(e\{x\}) < \ell n\, e$
$-\infty < \ell n(e\{x\}) < 1$

$$-\infty < \frac{1}{\ell n(e\{x\})} < 0 \quad \text{and} \quad 1 < \frac{1}{\ell n\, e\{x\}} < \infty$$

12. $$\lim_{n\to\infty}\left(\frac{1}{\log_x 2 \,.\, \log_x 4} + \frac{1}{\log_x 4 \,.\, \log_x 8} + \ldots\ldots + \frac{1}{\log_x 2^{n-1} \,.\, \log_x 2^n}\right)$$

$$= \lim_{n\to\infty}\frac{1}{\log_x 2}\left(\left(\frac{1}{\log_x 2} - \frac{1}{\log_x 4}\right) + \left(\frac{1}{\log_x 4} - \frac{1}{\log_x 8}\right) + \ldots\ldots + \left(\frac{1}{\log_x 2^{n-1}} - \frac{1}{\log_x 2^n}\right)\right)$$

$$= \lim_{n\to\infty}\frac{1}{\log_x 2}\left(\frac{1}{\log_x 2} - \frac{1}{\log_x 2^n}\right) = \log_2^2 x$$

13. $$\text{R.H.L.} = \lim_{x\to 3^+}(e^{x-3} - 1) = 0$$

$$\Rightarrow f(3) = 0 \Rightarrow \left[\frac{3}{a}\right] = 0 \qquad \Rightarrow 0 \le \frac{3}{a} < 1 \Rightarrow 0 \le 3 < a \Rightarrow a > 3 \quad \ldots\ldots\ldots\text{(i)}$$

$$\text{Also L.H.L.} = 0 \qquad \Rightarrow \lim_{x\to 3^-}\left[\frac{x}{a}\right] = 0$$

$$x < 3 \Rightarrow \frac{x}{a} < \frac{3}{a},\ a > 0 \Rightarrow \frac{3}{a} \le 1 \text{ i.e. } a \ge 3 \quad \ldots\ldots\text{(ii)}$$

$\therefore$ By (i) and (ii) $a \in (3, \infty)$

14. As $x \to \frac{\pi}{2}^-$ then $\tan^{-1} \tan x = x$

$x \to \frac{\pi}{2}^+$ $\tan^{-1}(\tan x) = \pi - x$ and $\sin(\pi - x) = \sin x$

$\ell = \lim\limits_{x \to \frac{\pi}{2}^-} \sqrt{\frac{\tan x - \sin x}{\tan x + \cos^2(\tan x)}}$ $\Rightarrow$ $\ell = \lim\limits_{x \to \frac{\pi}{2}^-} \sqrt{\frac{\sin x\,(1 - \cos x)}{\cos x[\tan x + \cos^2(\tan 2)}}$

$\ell = \lim\limits_{x \to \frac{\pi}{2}^-} \sqrt{\frac{\sin x\,(1 - \cos x)}{\sin x + \cos x.\cos^2(\tan x)}} = 1 = m$

$\therefore$ answer is $(x - 1)^2 = 0$

15. $f'(x) = 3\,(x - 3)^2$
for max. or min. $f'(x) = 0$
$\Rightarrow x = 3$ $\quad f''(x) = 6(x - 3)$ $\quad \Rightarrow f''(x) = 6(x - 3)$
$\Rightarrow f''(3) = 0$ $\quad \Rightarrow f'''(x) = 6$
since , $f'''(x) \neq 0$
hence f(x) neither max. nor min. at x = 3

16. For reservour A,

$\frac{dV_A}{dt} = -k_1 V_A$

$\Rightarrow \frac{dV_A}{V_A} = -k_1\,dt$ $\quad \Rightarrow \quad \ln V_A = -k_1 t + c$

initially t = 0, $V_A = V_{OA}$

$\therefore \ln V_{OA} = c$ $\quad \Rightarrow \quad \ln V_A = -k_1 t + \ln V_{OA}$

$\Rightarrow \ln\left(\frac{V_A}{V_{OA}}\right) = -k_1 t$ $\quad \Rightarrow \quad V_A = V_{OA}\,e^{-k_1 t}$ ..... (1)

For reservoir B,

$\frac{dV_B}{dt} = -k_2 V_B$

Similarly $V_B = V_{OB}\,e^{-k_2 t}$ ..... (2)

Given that at t = 0, $V_{OA} = 2V_{OB}$

At t = 1 hr. , $V_A = \frac{3}{2} V_B$

$\Rightarrow V_{OA}\,e^{-k_1} = \frac{3}{2} V_{OB}\,e^{-k_2}$

$\Rightarrow 2V_{OB}\,e^{-k_1} = \frac{3}{2} V_{OB}\,e^{-k_2}$ $\quad (\because \; V_{OA} = 2V_{OB})$

$\Rightarrow e^{(k_1 - k_2)} = \frac{4}{3}$ $\quad \Rightarrow \quad (k_1 - k_2) = \ln 4/3$ ..... (3)

After time t : $V_A = V_B$

$\Rightarrow V_{OA}\,e^{-k_1 t} = V_{OB}\,e^{-k_2 t}$

$\Rightarrow 2V_{OB}\,e^{-k_1 t} = V_{OB}\,e^{-k_2 t}$ $\quad (\because \; V_{OA} = 2V_{OB})$

$\Rightarrow 2 = e^{(k_1 - k_2)t}$

$\Rightarrow \ln 2 = (k_1 - k_2)t$ ..... (4)

from (3) and (4) we get $t = \frac{\ln 2}{\ln (4/3)}$

17. we have $2^m = 2^n + 56 \Rightarrow 2^m - 2^n = 56$
$\Rightarrow 2^n (2^{m-n} - 1) = 56 \qquad \Rightarrow 2^n (2^{m-n} - 1) = 8(8-1) \Rightarrow n = 3, \; m - n = 3$
$\therefore \; m = 6 \;, \; n = 3$

18. Graph is symmetric about $x = k$ if $f(k - x) = f(k + x)$
$\Rightarrow a(k-x)^3 + b(k-x)^2 + c(k-x) + d = a(k+x)^3 + b(k+x)^2 + c(k+x) + d$
$\Rightarrow 2ax^3 + (6ak^2 + 4bk + 2c)x = 0$ which will be true $\forall \; x \in R$
If $a = 0 \;\&\; 6ak^2 + 4bk + 2c = 0$

$\Rightarrow 4bk + 2c = 0 \Rightarrow k = -\frac{c}{2b} \qquad \therefore a + k = -\frac{c}{2b}$

19. f(x) is continuous at x = 0

$\therefore \; f(0) = \lim_{x \to 0} \frac{(128a + ax)^{1/8} - 2}{(32 + bx)^{1/5} - 2}$

as $x \to 0$ denominator $\to 0$. Thus for existance of limit numerator must also $\to 0$
$\therefore \; (128\,a)^{1/8} - 2 = 0 \Rightarrow (128a)^{1/8} = 2$
$\Rightarrow a = 2$

$\therefore \; f(0) = \lim_{x \to 0} \frac{(256 + 2x)^{1/8} - 2}{(32 + bx)^{1/5} - 2} = \frac{5}{32b} \quad \Rightarrow \quad b = \frac{5}{32 f(0)}$

$\therefore \; \frac{a}{b} = \frac{64}{5} f(0)$

20. $\because$ We have $f(x) = \frac{\ell n x}{x}$, $x > 0$

$\therefore \; f'(x) = \frac{1 - \ell n x}{x^2} < 0 \;\; \forall \, x > e \quad \Rightarrow \quad$ f(x) strictly decreases in (e, ∞)

Now $e < 202 < 303$
$\therefore \; f(303) < f(202)$

$\Rightarrow \frac{\ell n 303}{303} < \frac{\ell n 202}{202} \quad \Rightarrow \ell n\,(303)^{202} < \ell n\,(202)^{303} \quad \Rightarrow \quad (303)^{202} < (202)^{303}$

21. $\because \; 2f(\sin x) + f(\cos x) = x \;\forall\; x \in R$ .... (i)

$\therefore \; 2f\left(\sin\left(\frac{\pi}{2} - x\right)\right) + f\left(\cos\left(\frac{\pi}{2} - x\right)\right) = \frac{\pi}{2} - x$

$\Rightarrow 2f(\cos x) + f(\sin x) = \frac{\pi}{2} - x$ .... (ii)

by (i) & (ii) we get

$f(\sin x) = x - \frac{\pi}{6} \quad \Rightarrow \quad f(y) = \sin^{-1} y - \frac{\pi}{6}$

Thus $f(x) = \sin^{-1} x - \frac{\pi}{6}$

$\therefore \; D_f = [-1, 1] \;\&\; R_f = \left[\frac{-2\pi}{3}, \frac{\pi}{3}\right]$

22. $f(x) = (k-3)(k-4)\cos x + 2(k-4)x + \ell n2$

$\therefore f'(x) = -(k-3)(k-4)\sin x + 2(k-4)$

$= (k-4)[-(k-3)\sin x + 2]$

$\because$ f(x) have no critical points

$\therefore f'(x) = 0$ does not have any solution in R

$\Rightarrow k \neq 4$ and $2 - (k-3)\sin x = 0$ is not solvable in R

$\Rightarrow k \neq 4$ and $\left|\frac{2}{k-3}\right| > 1$

$\Rightarrow k \neq 4$ & $|k-3| < 2$

$\Rightarrow k \neq 4$ & $k-3 < 2$ and $k-3 > -2$ $\therefore k \in (1, 5) - \{4\}$

23. Given function can be re-defined as

$y > 0 \Rightarrow y = \frac{1}{2}x$; $x \geq 0$

$y < 0 \Rightarrow y = \frac{1}{10}x$; $x < 0$ $\Rightarrow$ $\therefore y = \begin{cases} \frac{x}{2}, & x \geq 0 \\ \frac{x}{10}, & x < 0 \end{cases}$

also $f(0+h) = f(0-h) = f(0) = 0$

$\Rightarrow$ f(x) is continuous at x = 0 but it is not diff. at x = 0 as $Rf'(0) = \frac{1}{2}$ & $Lf'(0) = \frac{1}{10}$

24. $\because 9y^2 = x^3$ $\therefore 18y\frac{dy}{dx} = 3x^2$ $\Rightarrow \frac{dy}{dx} = \frac{x^2}{6y}$

$\because$ normal makes equal intercepts on axis -

$\therefore \frac{-1}{dy/dx} = \pm 1$ $\Rightarrow \frac{dy}{dx} = \pm 1$ $\Rightarrow x^4 = 36y^2$

$\Rightarrow x^4 = 36\frac{x^3}{9} = 4x^3$ $\Rightarrow$ x = 0 , 4

$\therefore y = \pm 8/3$ $\Rightarrow$ $\therefore$ points are $\left(4, \pm\frac{8}{3}\right)$

25. Here, $f(x) = \cos|x| - 2ax + b = \cos x - 2ax + b$

$(\because \cos(-\theta) = \cos\theta$ for all real $\theta)$

$\Rightarrow$ $f'(x) = -\sin x - 2a.$

f(x) increases for all $x \in R$

If $f'(x) \geq 0$ for all $x \in R$ $\Rightarrow$ i.e. if $-\sin x - 2a \geq 0$

i.e. if $\sin x \leq -2a$ for all $x \in R$ $\Rightarrow$ i.e. if max. $\sin x \leq -2a$

$(\because$ max. $\sin x = 1)$ $\Rightarrow$ i.e. if $a \leq -\frac{1}{2}$

26. $y = \sin\frac{\pi}{2}\left[\frac{x}{2}\right] = 0$, $0 \leq x < 2$

$= 1$, $2 \leq x < 4$

$= 0$, $4 \leq x < 6$

$= -1$, $6 \leq x < 8$

$= 0$, $8 \leq x < 10$

= 1 , $10 \le x < 12$

Hence period is 8 and k = 4

27. $\because$ $2 < x^2 < 3$ $\Rightarrow$ $0 < x^2 - 2 < 1$ $\therefore$ $\{x^2\} = (x^2 - 2)$

$\because$ $x > 0$ $\Rightarrow$ $\sqrt{2} < x < \sqrt{3} \Rightarrow \frac{1}{\sqrt{3}} < \frac{1}{x} < \frac{1}{\sqrt{2}}$

$\therefore$ hence $\left\{\frac{1}{x}\right\} = \frac{1}{x}$ $\therefore$ $x^2 - 2 = \frac{1}{x}$

$(x + 1)(x^2 - x - 1) = 0$

$x \neq -1$

here $x = \frac{1+\sqrt{5}}{2}$ $(\because x > 0)$ only one solution

28. $f(x) = \frac{x^3}{3} - (m-3)\frac{x^2}{2} + mx$

$f'(x) = x^2 - (m-3)x + m \ge 0 \quad \forall x \in [0, \infty)$

Case-I $D \le 0 \Rightarrow m \in [1, 9]$ ....(i)

Case-II $D \ge 0 \Rightarrow m \in (-\infty, 1] \cup [9, \infty)$

$-\frac{b}{2a} \le 0 \Rightarrow m - 3 \le 0 \Rightarrow m \le 3$

$f(0) \ge 0 \Rightarrow m \ge 0$

$\Rightarrow m \in [0, 1]$ ....(ii)

from (i) and (ii) $m \in [0, 9]$ $\Rightarrow k = 9$

29. Here, $y^2 = x^3$ ....(1) $\Rightarrow$ $2y\frac{dy}{dx} = 3x^2$

slope of tangent at $(4m^2, 8m^3)$ is $\left(\frac{3x^2}{2y}\right)_{(4m^2, 8m^3)}$ = 3m and slope of normal at $(4m_1^2, 8m_1^3) = -\frac{1}{3m_1}$

Here $3m = -\frac{1}{3m_1}$

equation of the tangent at $(4m^2, 8m^3)$ is $y - 8m^3 = 3m(x - 4m^2)$

it passes through $(4m_1^2, 8m_1^3)$ $\Rightarrow$ $8m_1^3 - 8m^3 = 3m(4m_1^2 - 4m^2)$

$2(m_1^2 + m^2 + mm_1) = 3m(m_1 + m)$ $\Rightarrow$ $2(m_1^2) + 2m^2 + 2mm_1 = 3mm_1 + 3m^2$

$\Rightarrow$ $2m_1^2 = mm_1 + m^2$

$2\left(-\frac{1}{9m}\right)^2 = -\frac{1}{9} + m^2$ $\Rightarrow$ $\frac{2}{9(9m^2)} = \frac{-1+9m^2}{9}$

$\Rightarrow$ Let $9m^2 = t$ $\Rightarrow$ $\frac{2}{9t} = \frac{-1+t}{9}$

$\Rightarrow$ $t^2 - t - 2 = 0$ $\Rightarrow$ $(t-2)(t+1) = 0$ $t = 2$

30. $f(0^-) = 1$ $\Rightarrow$ and $f(0) = f(0^+) = 1$ $\Rightarrow$ Hence continuous at x = 0

$f\left(\frac{\pi}{2}^+\right) = f(\pi/2) = 3$ $\Rightarrow$ $f\left(\frac{\pi}{2}^-\right) = 1 + 1 + \sin\left(\frac{\pi}{2}^-\right) = 3$

Hence continuous at $x = \pi/2$ $\Rightarrow$ $f(1^+) = f(1) = 1 + 1 + \sin 1 = 2 + \sin 1$

$f(1^-) = 1 + 0 + \sin 1 = 1 + \sin 1$ $\Rightarrow$ Hence discontinuous at x = 1

SOLUTION TO PART TEST-05 TOPIC : INTEGRAL CALCULUS

Answers

1.	(2)	2.	(4)	3.	(2)	4.	(4)	5.	(4)	6.	(4)	7.	(3)
8.	(3)	9.	(4)	10.	(1)	11.	(3)	12.	(4)	13.	(3)	14.	(4)
15.	(4)	16.	(4)	17.	(3)	18.	(3)	19.	(2)	20.	(3)	21.	(4)
22.	(2)	23.	(1)	24.	(2)	25.	(1)	26.	(1)	27.	(1)	28.	(1)
29.	(4)	30.	(1)										

SOLUTIONS

1. $I = \int \frac{1}{\ln x} - \frac{1}{(\ln x)^2}\,dx = \int \frac{dx}{\ln x} - \int \frac{dx}{(\ln x)^2}$

Integrating by parts the first integral

$= \frac{1}{\ln x}\,.\,x - \int \frac{-1}{(\ln x)^2}\,.\,\frac{1}{x}\,x\,.\,dx - \int \frac{1}{(\ln x)^2}\,dx = \frac{x}{\ln x} + c$

2. $\int \frac{dx}{\sqrt{1+\operatorname{cosec}^2 x}} = \int \frac{\sin x\,dx}{\sqrt{\sin^2 x + 1}} = \int \frac{\sin x\,dx}{\sqrt{2-\cos^2 x}}$

$= -\int \frac{dt}{\sqrt{2-t^2}}$, where $t = \cos x$ $= \cos^{-1}\frac{t}{\sqrt{2}} + c = \cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) + c$

3. Let $I_1 = \int \frac{1}{x+x^5}\,dx$ and $I_2 = \int \frac{x^4}{x+x^5}\,dx$

Now $I_1 + I_2 = \int \frac{dx}{x} = \ln|x| + c \Rightarrow I_2 = \ln|x| - f(x) + c$

4. $I = \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx)\,dx - 2\int_{-\pi}^{\pi} (\cos ax.\,\sin bx)\,dx$

$= 2\int_0^{\pi} (\cos^2 ax + \sin^2 bx)\,dx - 0 = \int_0^{\pi} (1+\cos 2ax + 1 - \cos 2bx)\,dx = 2\pi + 0 = 2\pi$

5. Since $|f(x)| \geq 0$ and $\int_a^b |f(x)|\,dx = 0$

$\Rightarrow$ $|f(x)| = 0$ over the interval (a, b) $\Rightarrow$ $(f(x))^2 = 0$ on (a, b)

$\therefore$ statement 2 is true

$\because$ $\sqrt{x^2} = |x| \Rightarrow A = \int_{-3}^{3} |x|\,dx = 2\int_{3}^{3} |x|\,dx$

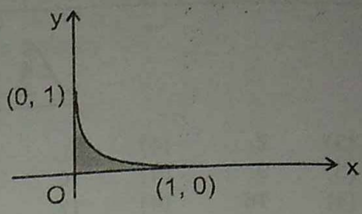
$\Rightarrow$ ($\because$ $|x|$ is even) $\Rightarrow$ $2\int_0^3 x\,dx = 2\left(\frac{x^2}{2}\right)_0^3 = 9$

$\therefore$ statement 1 is false

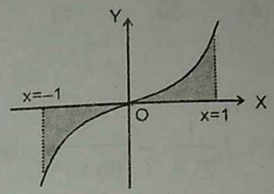
6. $A = \int_0^1 y\,dx = \int_0^1 (1-\sqrt{x})^2\,dx$

$= \int_0^1 (1 - 2\sqrt{x} + x)\,dx$

$= 1 - \frac{4}{3} + \frac{1}{2} = \frac{1}{6}$



7. $y = x\,|\,x\,| = \begin{cases} x^2 & x \ge 0 \\ -x^2 & x < 0 \end{cases}$



required area $= \left|\int_{-1}^{0} -x^2 dx\right| + \left|\int_0^1 x^2 dx\right| = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

8. The general equation of non-vertical lines in a plane is $ax + by = 1$, $b \neq 0$

$\Rightarrow a + b\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{a}{b} \Rightarrow \frac{d^2y}{dx^2} = 0$

9. $y^2 = 2c\left(x + \sqrt{c}\right)$

$\Rightarrow 2yy_1 = 2c \quad \Rightarrow \quad c = yy_1 \Rightarrow (y^2 - 2xyy_1)^2 = (2(yy_1)^{3/2})^2 \quad = 4(yy_1)^3$

The order is 1 and degree is 3

10. $\frac{dV}{dt} = -ks$

$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = -k\,(4\pi r^2) \quad \Rightarrow \quad \frac{dr}{dt} = -k \Rightarrow r = c - kt$

$r(0) = 3 \Rightarrow c = 3$, $r = 3 - kt \quad \Rightarrow \quad r(1) = 2 \Rightarrow k = 1$, $r = 3 - t \qquad r(3) = 0$

11. $f(x) = (1 + a)x + b$

$a = \int_0^1 t\,f(t)dt = \frac{1+a}{3} + \frac{b}{2} \quad \Rightarrow \quad b = \int_0^1 t^2 + f(t)dt = \frac{1+a}{4} + \frac{b}{3}$

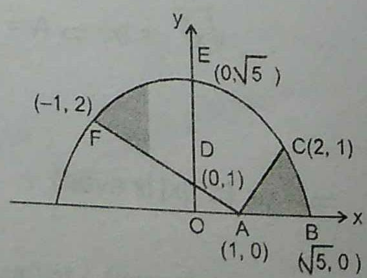
$\Rightarrow \quad 4a - 3b = 2$, $8b - 3a = 3 \quad \Rightarrow \quad a = \frac{25}{23}$, $b = \frac{18}{23}$

$\Rightarrow \quad \int_0^1 f(x)dx = \frac{1+a}{2} + b = \frac{42}{23}$

12. $L = \lim_{n\to\infty} \frac{\left(\frac{1}{n}\sum_{r=1}^{n}\sqrt{\frac{r}{n}}\right)\left(\frac{1}{n}\sum_{r=1}^{n}\sqrt{\frac{n}{r}}\right)}{\frac{1}{n}\sum_{r=1}^{n}\frac{r}{n}} = \frac{\int_0^1 \sqrt{x}dx . \int_0^1 \frac{1}{\sqrt{x}}dx}{\int_0^1 x\,dx} = \frac{2}{3} \,.\, 2 \,.\, 2 = \frac{8}{3}$

13. The shaded areas are congruent
Area = The area ACEF

$= \frac{1}{4}$ (area of circle) $- \Delta OAD = \frac{5\pi}{4} - \frac{1}{2} = \frac{5\pi - 2}{4}$



14. $x^2 + 2y^2 - y = c$

$\Rightarrow 2x + 4y \cdot \frac{dy}{dx} - \frac{dy}{dx} = 0 \Rightarrow 2x = -(4y-1)\frac{dy}{dx}$

replace $\frac{dy}{dx}$ by $-\frac{1}{\left(\frac{dy}{dx}\right)}$ $\Rightarrow$ $\frac{2xdy}{dx} = 4y - 1$ $\Rightarrow$ $\int \frac{4dy}{4y-1} = \int 2\frac{dx}{x}$

$\ell n(4y-1) = \ell nx^2 + \ell nc_1$ $\Rightarrow$ $4y - 1 = c_1x^2$

$y = cx^2 + \frac{1}{4}$, where $4c = c_1$ $\Rightarrow$ $y = cx^2 + 1/4$, where $c = c_1/4$

15. **Statement-1 :** $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx = \int e^x\left(\frac{1-\sin x}{2\sin^2\frac{x}{2}}\right)dx = \int e^x\left(\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right)dx$

$= -\int e^x\left(\cot\frac{x}{2} + \left(-\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}\right)\right)dx = -e^x\cot\frac{x}{2} + c$

$\therefore$ Statement-1 is false **Statmenent-2 :** is true

16. Let $x = t + 17$

$dx = dt$

for $x = 17, t = 0$

$x = 18 \quad t = 1$

$\Rightarrow \quad I = \int_0^1 \frac{e^{-t-17}}{t-2}dt = e^{-18}\int_0^1 \frac{e^{-t+1}}{t-2}dt$

$\therefore \quad \int_0^a f(x)dx = \int_0^a f(a-x)dx$

$\therefore \quad I = e^{-18}\int_0^1 \frac{e^t}{-(t+1)}dt = -\lambda e^{-18}$

17. $\because \quad [x] \in I \quad \& \quad [x + I] = [x] + I$

$\therefore \quad [x - 2[x]] = [x] - 2[x] = -[x]$

hence $I = \int_0^2 (x-[x])^{[2x]}dx$

$I = \int_0^{1/2} x^0dx + \int_{1/2}^1 x^1dx + \int_1^{3/2}(x-1)^2dx + \int_{3/2}^2 (x-1)^3dx = [x]_0^{1/2} + \left(\frac{x^2}{2}\right)_{1/2}^1 + \left[\frac{(x-1)^3}{3}\right]_1^{3/2} + \left(\frac{[x-1]^4}{4}\right)_{3/2}^2 = \frac{221}{192}$

18. Let $t = x^{\text{Sinx}}$

$\Rightarrow \log t = \sin x . \log x \quad \Rightarrow \quad \frac{1}{t}dt = \cos x \log x + \frac{\sin x}{x} \quad \Rightarrow \quad dt = x^{\text{Sinx}}. \log x^{\text{Cosx}} + \sin x.\ x^{\text{Sinx}-1}$

$\therefore I = \int dt = t + c$

19. Any line at a distance of '3' units from origin is

$x\cos\theta + y\sin\theta = 3$ ....(1)

differentiating

$\cos\theta + \sin\theta \dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\cot\theta$

from equation (1) $x \cot\theta + y = 3 \operatorname{cosec}\theta$

$$-\frac{dy}{dx}x + y = 3\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\left(y - \frac{xdy}{dx}\right)^2 = 9\left[1+\left(\frac{dy}{dx}\right)^2\right]$$

20. $y = \displaystyle\int \frac{\sin[(x+\alpha)-\alpha]}{\sin[x+\alpha]}dx$

$\Rightarrow \quad y = \displaystyle\int \frac{\sin(x+\alpha).\cos\alpha}{\sin(x+\alpha)}dx - \int \frac{\cos(x+\alpha).\sin\alpha}{\sin(x+\alpha)}dx$

$y = x\cos\alpha - \sin\alpha \,\ell n\,|\sin(x+\alpha)| + c$

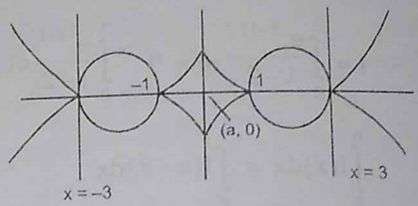
$\Rightarrow \quad A = \cos\alpha, \quad B = \sin\alpha$

Which lies on both $x\cos\alpha + y\sin\alpha = 1$ and $x^2 + y^2 = 1$

21. $y = x^2 - 4x + 3 = (x-1)(x-3)$

graph of $|y| = |x^2 - 4x + 3|$

Required Area



$$= \left[\int_0^1 (x^2 - 4x + 3)\,dx + \int_1^3 -(x^2 - 4x + 3)\,dx\right] \times 4$$

$$= 4\left[\left(\frac{x^3}{3} - \frac{4x^2}{2} + 3x\right)_0^1 - \left(\frac{x^3}{3} - \frac{4x^2}{2} + 3x\right)_1^3\right] = \frac{32}{3}$$

22. $I = \displaystyle\int \frac{(e^{2x}-1)\,dx}{e^{4x}\sqrt{2-2e^{-2x}+e^{-4x}}} = \int \frac{(e^{-2x}-e^{-4x})\,dx}{\sqrt{2-2e^{-2x}+e^{-4x}}}$

Put $t^2 = 2 - 2e^{-2x} + e^{-4x}$

$2t\,dt = (4e^{-2x} - 4e^{-4x})\,dx$

$\therefore \quad I = \displaystyle\int \frac{1/2tdt}{t} = \frac{1}{2}t + c \quad \therefore \quad I = \frac{1}{2}\sqrt{2-2e^{-2x}+e^{-4x}} + c$

23. $6\displaystyle\int_0^1 (a + bx + cx^2)\,dx = 6\left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1 = 6a + 3b + 2c$

also, $f(0) = a$, $f(1/2) = a + \dfrac{b}{2} + \dfrac{c}{4}$ & $f(1) = a + b + c$

Hence $6\displaystyle\int_0^1 f(x)\,dx = f(0) + 4f\left(\frac{1}{2}\right) + f(1)$

24. Differentiating

$xf(x) = 2x + x^2f(x)$

$\Rightarrow x(1-x)f(x) = 2x \quad \Rightarrow \quad (1-x)\,f(x) = 2 \;$ if $x \neq 0$

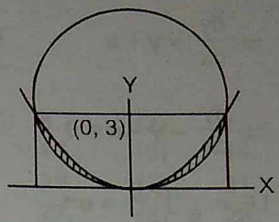
$f(x) = \dfrac{2}{1-x} \quad$ if $x \neq 1$

Domain of f(x) is R – {0, 1}

25. $x^2 + y^2 - 6y \le 0$
centre (0, 3) radius 3
$x^2 = 3y$

$$\therefore \text{ area} = \frac{9\pi}{2} - \left(18 - 2\int_0^3 \frac{x^2}{3}\,dx\right) = \frac{9\pi}{2} - 18 + 2\left.\frac{x^3}{9}\right|_0^3$$

$$= \frac{9\pi}{2} - 18 + 6 = \frac{9\pi}{2} - 12$$



26. $$\int_0^{\sqrt{3}} \frac{\sin^{-1}\left(\frac{2x}{1+x^2}\right)}{1+x^2}\,dx = \int_0^{\pi/3} \sin^{-1}(\sin 2\theta)\,d\theta \qquad (x = \tan\theta)$$

$$= \int_0^{\pi/4} 2\theta\,dx + \int_{\pi/4}^{\pi/3} (\pi - 2\theta)\,d\theta = \theta^2\Big|_0^{\pi/4} + \pi\left(\frac{\pi}{3} - \frac{\pi}{4}\right) - \theta^2\Big|_{\pi/4}^{\pi/3} = \frac{7\pi^2}{72}$$

27. $$\int_0^{\sqrt{3}} \frac{\sin^{-1}\left(\frac{2x}{1+x^2}\right)}{1+x^2}\,dx = \int_0^{\pi/3} \sin^{-1}(\sin 2\theta)\,d\theta \qquad (x = \tan\theta)$$

$$= \int_0^{\pi/4} 2\theta\,dx + \int_{\pi/4}^{\pi/3} (\pi - 2\theta)\,d\theta = \theta^2\Big|_0^{\pi/4} + \pi\left(\frac{\pi}{3} - \frac{\pi}{4}\right) - \theta^2\Big|_{\pi/4}^{\pi/3} = \frac{7\pi^2}{72}$$

28. Given $f'(1) = -\frac{1}{\sqrt{3}}$, $f'(2) = -\sqrt{3}$ and $f'(3) = -1$

$$\int_2^3 f'(x)\,f''(x)\,dx + \int_1^3 f''(x)\,dx = \left[\frac{(f'(x))^2}{2}\right]_2^3 + [f'(x)]_1^3 = \frac{(f'(3))^2 - (f'(2))^2}{2} + f'(3) - f'(1) = -2 + \frac{1}{\sqrt{3}}$$

29. $\sec^2 y \frac{dy}{dx} + 2x\tan y = x^3 \quad \Rightarrow \quad \frac{d}{dx}(\tan y) + 2x\tan y = x^3$

Linear D. equation with I.F. $= e^{\int 2x dx} = e^{x^2} \quad \Rightarrow \quad$ solution is $\tan y\, e^{x^2} = \int x^3.e^{x^2}dx$

$\tan y.\, e^{x^2} = \frac{1}{2}\int te^t dt$, let $x^2 = t \quad \Rightarrow \quad 2x\,dx = dt$

$\tan y.\, e^{x^2} = \frac{1}{2}(t-1)\,e^t + c \quad \Rightarrow \quad \tan y = \frac{1}{2}(x^2 - 1) + c\,e^{-x^2}$

$x = 0$, $y = 0 \quad \Rightarrow \quad c = \frac{1}{2}$

$\Rightarrow \tan y = \frac{1}{2}\left[x^2 - 1 + e^{-x^2}\right] \quad \Rightarrow \quad$ at $x = 1$, $\tan y = \frac{1}{2e} \quad \Rightarrow y = \tan^{-1}(1/2e)$

30. $\frac{dy}{dx} = y + \int_0^1 y\,dx$

$\Rightarrow \frac{dy}{dx} = y + a$ let $\int_0^1 y\,dx = a$ $\int_0^1 y\,dx = a$

$\Rightarrow \frac{dy}{dx} - y - a = 0$

I.F. $= e^{-\int 1.dx} = e^{-x}$, $\Rightarrow$ Now $e^{-x}\frac{dy}{dx} - ye^{-x} - ae^{-x} = 0$

$\Rightarrow ye^{-x} + ae^{-x} + c = 0$ $\Rightarrow y + a + ce^{x} = 0$

$a = \int_0^1 y\,dx = -\int_0^1 (a + ce^{x})dx = -a(1-0) - c(e^{x})\Big|_0^1 = -a - c(e-1)$

$\Rightarrow a = \frac{(1-e)c}{2}$ $\Rightarrow y + \left(\frac{(1-e)+2e^{x}}{2}\right)c = 0$

$\Rightarrow y = 1; x = 0$ $\Rightarrow c = \frac{2}{e-3}$ $\Rightarrow y = \left(\frac{2e^{x} - e + 1}{3-e}\right)$

SOLUTION TO PART TEST-6 TOPIC : ALGEBRA-2 & GEOMETRY (3-D)

Answers

1.	(1)	2.	(4)	3.	(1)	4.	(4)	5.	(2)	6.	(2)	7.	(1)
8.	(1)	9.	(4)	10.	(2)	11.	(2)	12.	(3)	13.	(3)	14.	(2)
15.	(2)	16.	(2)	17.	(1)	18.	(2)	19.	(3)	20.	(3)	21.	(3)
22.	(4)	23.	(3)	24.	(1)	25.	(4)	26.	(1)	27.	(3)	28.	(2)
29.	(3)	30.	(3)										

SOLUTIONS

1. $P(W) = \frac{a}{a+b}$, $P(2) = \frac{b}{a+b}$

P (A wins) = P (W) + $(P(2))^2$ P(W) + $(P(2))^4$ P(W) + ...

$$= \frac{P(W)}{1-(P(B))^2} = \frac{\frac{a}{(a+b)}}{1-\left(\frac{b}{a+b}\right)^2} = \frac{a+b}{a+2b}$$

and P (B wins) = 1 – P (A wins) = $\frac{b}{a+2b}$

∴ According to the given condition

$$\left(\frac{a+b}{a+2b}\right) = 2.\frac{b}{(a+2b)}$$

$\Rightarrow$ a = b $\Rightarrow$ $\frac{a}{b} = \frac{1}{1}$

2. Sum of vectors

∴ Unit vectors || to this sum = $\frac{(b+2)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(b+2)^2+36+4}} = \frac{(b+2)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{b^2+4b+44}} = \vec{r}$ (say).

Now $(\hat{i}+\hat{j}+\hat{k}).\vec{r} = 1$

$\Rightarrow$ (b + 2) + 6 – 2 = $\sqrt{b^2+4b+44}$ $\Rightarrow (b+6)^2 = b^2 + 4b + 44 \Rightarrow b = 1$

3. $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{1}{2}(\vec{b}+\vec{c})$

$\Rightarrow$ $(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = \frac{1}{2}(\vec{b}+\vec{c})$ $\Rightarrow$ $(\vec{a}.\vec{c}) = \frac{1}{2}$ and $\vec{a}.\vec{b} = \frac{-1}{2}$

$(\because \vec{b} \neq \lambda\vec{c})$ (λ : real parameter)

$\Rightarrow$ required sum $\frac{\pi}{3} + \frac{2\pi}{3} = \pi$

4. Using
Distance between centre and given
Plane = radius of sphere

radius of sphere = $\frac{|(3\hat{i}+6\hat{j}-4\hat{k}).(2\hat{i}-2\hat{j}-\hat{k})-10|}{\sqrt{4+4+1}} = 4$

∴ Required sphere has equation

$|\vec{r}-(3\hat{i}+6\hat{j}-4\hat{k})|=4$

5. Let required plane

a(x – 2) + b(y – 1) + c(z – 4) = 0

Now we have

3a + (– b) + c = 0

5a + b + 3c = 0

$\therefore \frac{a}{-4}=\frac{b}{-4}=\frac{c}{8}=k$ (let)

∴ required plane is x + y – 2z = – 5

6. $AL = \frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$

Now let L(3r + 6, 2r + 7, –2r + 7)

∴ DR's of PL

(3r + 5, 2r + 5, – 2r + 4)

∴ PL perpendicular to AL

∴ 3(3r + 5) + 2(2r + 5) – 2(–2r + 4) = 0

⇒ 17r + 17 = 0

∴ r = – 1 ∴ L (3, 5, 9)

∴ Required Area = $\frac{1}{2}\left|\overrightarrow{AL}\times\overrightarrow{PL}\right| = \frac{1}{2}\sqrt{17}.7$

7. Applying $R_3 \to R_3 - R_1$
$R_2 \to R_2 - R_1$

$$\Delta = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin(B+A)\sin(B-A) & \frac{\sin(A-B)}{\sin A \sin B} & 0 \\ \sin(C+A)\sin(C-A) & \frac{\sin(A-C)}{\sin A \sin C} & 0 \end{vmatrix} \Rightarrow \Delta = \sin(A-B)\sin(A-C)\begin{vmatrix} -\sin C & \frac{1}{\sin A \sin B} \\ -\sin B & \frac{1}{\sin A \sin C}\end{vmatrix} = 0$$

8. Given system will be consistent if

$$\Delta = \begin{vmatrix} 1 & 1 & -3 \\ 1+\lambda & 2+\lambda & -8 \\ 1 & -(1+\lambda) & 2+\lambda \end{vmatrix} = 0 \Rightarrow \lambda = -\frac{5}{3} \text{ or } \lambda = 1$$

9. Adding all we get

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 3$

subtracting first equation from it

we get $\frac{2z^2}{c^2} = 2$ ∴ z = ± c

similarly y = ± b

and x = ± a

∴ Given system has eight solutions.

10. Given A′ = A, B′ = B

(AB – BA)′ = (AB)′ – (BA)′

= B′A′ – A′B′ = BA – AB = – (AB – BA)

⇒ Hence AB – BA is a skew symmetric matrix.

11. Let A : minimum of the chosen number is 3

B : maximum of chosen number is 7

$P(1) = P(\text{choosing 3 and two other number from 4 to 10}) = \frac{^7C_2}{^{10}C_3} = \frac{7}{40}$

$P(2) = P(\text{choosing 7 and two other number from 1 to 6}) = \frac{^6C_2}{^{10}C_3} = \frac{1}{8}$

$P(A \cap B) = P(\text{choosing 3 and 7 and one other number from 4 to 6}) = \frac{3}{^{10}C_3} = \frac{1}{40}$

$P(A \cup B) = P(1) + P(2) - P(A \cap B) = \frac{7}{40} + \frac{1}{8} - \frac{1}{40} = \frac{11}{40}$

12. $0.8 = P(1) + P(2) - P(A \cap B)$
$= P(1) + P(2) - P(1) . P(2)$
($\because$ A, B are independent)

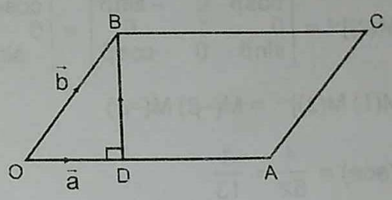
$\therefore 0.8 = 0.3 + P(2) - 0.3 (P(2)) \Rightarrow \qquad P(2) = \frac{5}{7}$

13. Let $\overrightarrow{OD} = t\vec{a}$

$\therefore \quad \overrightarrow{DB} = \vec{b} - t\vec{a}$

$\therefore \quad \overrightarrow{DB} \perp \overrightarrow{OA}$

$\therefore \quad (\vec{b} - t\vec{a}).\vec{a} = 0$



$t = \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}$

$\overrightarrow{DB} = \vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$

14. $\Delta = (x\,y\,z)^n \begin{vmatrix} 1 & x^2 & x^4 \\ 1 & y^2 & y^4 \\ 1 & z^2 & z^4 \end{vmatrix} = (x\,y\,z)^n (x^2 - y^2)(y^2 - z^2)(z^2 - x^2)$

$\therefore$ for required result
$n = -4$

15. (2) $P(B/A \cup B^c) = \frac{P[B \cap (A \cup B^c)]}{P(A \cup B^c)}$

Now, $P[B \cap (A \cup B^c)] = P[(B \cap A) \cup (B \cap B^c)] = P[(B \cap A) \cup \phi]$
$P = P(A \cap B) = P(A) - P(A \cap B^C) = 0.7 - 0.5 = 0.2$

Again $P(A \cup B^c) = P(A) + P(B^C) - P(A \cap B^C) = 0.7 + (1 - 0.4) - 0.5 = 0.8$

$\therefore \quad P(B/A \cup B^c) = \frac{0.2}{0.8} = 0.25$

16. $\left| \frac{\ell(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)}{\sqrt{\ell^2 + m^2 + n^2}} \right| = |\ell(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$

17. We observe that the line given in option (1) passes through (1, 2,3). Also it is perpendicular to the normal of the plane $2x + 3y + z = 5$

18. We have

$|\vec{a}| = 1 \;, |\vec{b}| = 1 \;, |\vec{c}| = 2$

Also $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = 0$

$(\vec{a}.\vec{c})\vec{a} - (\vec{a}.\vec{a})\vec{c} + \vec{b} = 0$

$(\vec{a}.\vec{c})\vec{a} - \vec{c} + \vec{b} = \vec{0}$ $\quad [\because \vec{a}.\vec{a} = |\vec{a}|^2 = 1]$

$|(\vec{a}.\vec{c})\vec{a} - \vec{c}| = |-\vec{b}|$ $\Rightarrow$ $|(\vec{a}.\vec{c})\vec{a}|^2 + |\vec{c}|^2 - 2((\vec{a}.\vec{c})(\vec{a}.\vec{c})) = |\vec{b}|^2$

$|(\vec{a}.\vec{c})\vec{a}|^2(|\vec{a}|^2 - 2) + |\vec{c}|^2 = |\vec{b}|^2 \Rightarrow -(a.c)^2 + 4 = 1$ $\quad [\because |\vec{a}|^2 = 1 \ \& \ |\vec{c}|^2 = 4]$

$(\vec{a}.\vec{c})^2 = 3$ $\Rightarrow$ $\vec{a}.\vec{c} = \pm\sqrt{3}$

acute angle $\theta = 30°$

19. $[M(1)\,M(2)]^{-1} = M(2)^{-1}\,M(1)^{-1}$

Now $[M(1)]^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} = M(-\alpha)$

$[M(2)]^{-1} = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \equiv \begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) \\ 0 & 1 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix} = M(-\beta)$

$[M(1)\,M(2)]^{-1} = M(-\beta)\,M(-\alpha)$

20. $P(\text{ace}) = \dfrac{4}{52} = \dfrac{1}{13}$

$\Rightarrow P(\text{king}) = \dfrac{4}{52} = \dfrac{1}{13}$

$P(\text{2 aces and one king}) = {}^3C_2 \times \left(\dfrac{1}{13}\right)^2 \times \left(\dfrac{1}{13}\right) = \dfrac{3}{(13)^3}$

21. Equation of a plane through the line of intersection of given plane is -

$ax + by + cz + d + \lambda(a'x + b'y + c'z + d') = 0$

$\Rightarrow (a+\lambda a')x + (b+\lambda b')y + (c+\lambda c')z + (d+\lambda d') = 0$

It is parallel to $y = 0$, $z = 0$ i.e. x-axis whose direction ratios are 1, 0, 0

$1(a+\lambda a') + 0(b+\lambda b') + 0(c+\lambda c') = 0$

$\Rightarrow \lambda = \dfrac{-a}{a'}$

Hence the required plane is -

$y(a'b - ab') + z(a'c - ac') + (a'd - ad') = 0$

or $y(ab' - a'b) + z(ac' - a'c) + (ad' - a'd) = 0$

22. Let A be the first term and R be the common ratio of the given GP then

$a = AR^{p-1}$, $b = AR^{q-1}$, $c = AR^{r-1}$

$\Rightarrow a^{q-r}\,b^{r-p}\,c^{p-q} = A^{\circ}R^{\circ} = 1$ $\Rightarrow (q-r)\log a + (r-p)\log b + (p-q)\log c = 0$ [taking log on both sides]

$\Rightarrow \vec{u}.\vec{v} = 0 \Rightarrow \vec{u} \perp \vec{v}$ Hence angle = 90°

23. Given $(ax - b)^2 + (bx - c)^2 + (cx - d)^2 \le 0$

$\Rightarrow ax - b = bx - c = cx - d = 0$

$\Rightarrow \quad \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = x$

$\therefore \quad b^2 = ac$ or $2 \log b = \log a + \log c$

$$\text{Now } \Delta = \begin{vmatrix} 1 & 1 & \log a \\ 1 & 2 & \log b \\ 1 & 3 & \log c \end{vmatrix} = 0$$

using $R_1 \to R_1 + R_3 - 2R_2$

24. $(A + B)^m = {}^mC_0 A^m + {}^mC_1 A^{m-1} B + ... + {}^mC_m B^m \quad \Rightarrow \quad AB = BA$

25. Let $\vec{c} = \lambda\vec{a} + \mu\vec{b} + \nu\, \vec{a}\times\vec{b}$

then $\vec{a}.\vec{c} = \lambda\vec{a}.\vec{a} \quad \therefore \quad \lambda = \frac{1}{4\sqrt{2}}$

$\vec{b}.\vec{c} = \mu\vec{b}.\vec{b} \quad \therefore \quad \mu = \frac{1}{6}$

and $(\vec{a}\times\vec{b}).\vec{c} = \nu\,(\vec{a}\times\vec{b})^2$

$\therefore \quad \nu = \frac{V}{144} \quad$ where $V = (\vec{a}\times\vec{b}).\vec{c}$

Also $\vec{c}.\vec{c} = \lambda\,\vec{a}.\vec{c} + \mu\,\vec{b}.\vec{c} + \nu V$

$1 = \lambda\, 2\sqrt{2} + \mu\,.\,\frac{3}{2} + \nu\,.\,144\nu = \frac{1}{2} + \frac{1}{4} + 144\nu^2$

$\nu^2 = \frac{1}{4\times144} \quad \nu = \pm\frac{1}{24}$

$\therefore$ there are two values of vector $\vec{c}$ and $V = \pm 6$

$\therefore$ volume of tetrahedron = 1

26. $A^T = BCD$, $B^T = CDA$, $C^T = DAB$ and $D^T = ABC$

Let orders of A, B, C and D be $\ell \times m$, $m \times n$, $n \times p$ and $p \times \ell$

$\therefore$ S is $\ell \times \ell$ is a square matrix.

$S = ABCD$

$S^T = D^T C^T B^T A^T = ABCD\, ABC\, DA\, BCD = (ABCD)^3 = S^3$

thus $S^T = S^3 \quad \Rightarrow \quad S = (S^T)^3 = S^9$

27. Split the digits into pairs viz : (0, 1), (1, 2) , ... (8, 9)

Disjoint pairs out of these are (0, 1), (2, 3), (4, 5), (6, 7), (8, 9)

(there is no other set of 5 disjoints pairs)

Now two cases are

(a) When the pair (0, 1) is not used for first and the last place.

The number of ways = $4 \times 4! \times 2^5$

(b) When the pair (0, 1) is used for first and last place.

The number of ways = $1 \times 4! \times 2^4$

$\therefore$ Total number of favourable cases = $9 \times 4! \times 2^4$

$$\therefore \quad \text{Probability} = \frac{9 \times 4! \times 2^4}{9 \times 9!} = \frac{1}{945}$$

28. Let $D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$C_2 \to C_2 - \frac{a_{12}}{a_{11}} C_1 \qquad C_3 \to C_3 - \frac{a_{13}}{a_{11}} C_1$$

$$D = \begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & \left(a_{22} - \frac{a_{12}}{a_{11}} \times a_{21}\right) & \left(a_{23} - \frac{a_{13}}{a_{11}} a_{21}\right) \\ a_{31} & \left(a_{32} - \frac{a_{12}}{a_{11}} \times a_{31}\right) & \left(a_{32} - \frac{a_{13}}{a_{11}} \times a_{31}\right) \end{vmatrix}$$

so minimum value of D = –4

29. Total number of point = 6 × 6 = 36

favorable points will lie on the line x + y = 4

so favorable points are (1, 3), (2, 2) & (3, 1)

$$\text{so probability} = \frac{3}{36} = \frac{1}{12}$$

30. Number of ways for which 1 is the highest common factor is = ${}^5C_0 + {}^5C_1 + {}^5C_2 = 16$

Number of ways for which 3 is the highest common factor is = $2\left({}^4C_0 + {}^4C_1 + \frac{4!}{2!(2!)^2}\right) = 16$

Number of ways for which 5 is the highest common factor is = ${}^4C_0 + {}^4C_1 + \frac{4!}{2!(2!)^2} = 8$

Total number of ways in which N can be resolved into product of 2 factors = $\frac{6 \times 8 \times 3 \times 5 \times 2}{2} = 720$

$$\therefore \quad \text{Probability} = \frac{16 + 16 + 8}{720} = \frac{1}{18}$$

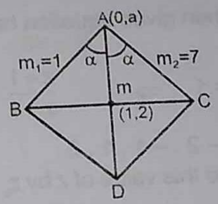
SOLUTIONS TO FULL SYLLABUS TEST- 01

Answers

1.	(4)	2.	(3)	3.	(3)	4.	(1)	5.	(3)	6.	(2)	7.	(4)
8.	(2)	9.	(2)	10.	(3)	11.	(1)	12.	(3)	13.	(4)	14.	(1)
15.	(3)	16.	(3)	17.	(3)	18.	(2)	19.	(1)	20.	(3)	21.	(1)
22.	(2)	23.	(3)	24.	(2)	25.	(1)	26.	(4)	27.	(3)	28.	(3)
29.	(4)	30.	(1)										

SOLUTIONS

1. $\because \tan\alpha = \dfrac{1-m}{1+m} = \dfrac{m-7}{1+7m}$



$\Rightarrow 2m^2 - 3m - 2 = 0 \quad \Rightarrow m = 2, m = -\dfrac{1}{2}$

$\therefore\ 2 - a = 2 \ \text{ or } \ 2 - a = -\dfrac{1}{2} \quad \Rightarrow a = \dfrac{5}{2}$

2.
B _ _ _ _ _ = 5!
C _ _ _ _ _ = 5!
E _ _ _ _ _ = 5!
J _ _ _ _ _ = 5!
O _ _ _ _ _ = 5!
T B _ _ _ _ = 4!
T C _ _ _ _ = 4!
T E _ _ _ _ = 4!
T J _ _ _ _ = 4!
T O B _ _ _ = 3!
T O C _ _ _ = 3!
T O E _ _ _ = 3!
T O J B _ _ = 2!
T O J C B E = 717

3.
$\alpha + \beta = 6P_1 \qquad \alpha\beta = 2$ .... (1)
$\beta + \gamma = 6P_2 \qquad \beta\gamma = 3$ .... (2)
$\alpha + \gamma = 6P_3 \qquad \alpha\gamma = 6$ .... (3)

from equation (1), (2) & (3)

$\alpha\beta\gamma = 6$

so $\gamma = 3,\ \alpha = 2,\ \beta = 1$

$$C = \left(\frac{\alpha+\beta+\gamma}{3}, \frac{1}{3}\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)\right) = \left(\frac{6}{3}, \frac{1}{3}\left(\frac{1}{2}+1+\frac{1}{3}\right)\right) = \left(2, \frac{11}{18}\right)$$

4. AB be the normal chord

$A = (at^2, 2at) \quad B(at_1^2, 2at_1)$

$t_1 = -t - \dfrac{2}{t}$

$AB^2 = [a^2(t^2 - t_1^2)^2] + 4a^2(t - t_1)^2$

$= a^2(t-t_1)^2\,[(t+t_1)^2+4] \;=\; a^2\left(t+t+\frac{2}{t}\right)\left[\frac{4}{t^2}+4\right] = \frac{16a^2(1+t^2)^3}{t^4}$

$$\frac{d(AB^2)}{dt} = \frac{32a^2(1+t^2)^2(t^2-2)}{t^5}$$

for $\frac{d(AB^2)}{dt} = 0 \Rightarrow t = \sqrt{2}$ for which AB^2 is in minimum

$$AB_{min} = \frac{4a}{2}(1+2)^{3/2} = 2a\sqrt{27}$$

5. Statement-2 is false, as axis of parabola is normal to parabola which pass through the focus however normal other than axis never passes through focus.
Statement-1 is correct as $x - y - 5 = 0$ passes through focus $(3, -2)$. Hence it cannot be normal.

6. If $z \neq 1$ then given equation can be written as

$$\left(\frac{z+1}{z-1}\right)^5 = 1 \quad \Rightarrow \quad \frac{z+1}{z-1} = e^{i\frac{2k\pi}{5}}$$

where $k = -2, -1, 1, 2$
If we denote this value of z by z_k then

$$z_k = \frac{e^{i\frac{2k\pi}{5}}+1}{e^{i\frac{2k\pi}{5}}-1} = \frac{e^{i\frac{k\pi}{5}}+e^{-k\frac{i\pi}{5}}}{e^{i\frac{k\pi}{5}}-e^{-i\frac{k\pi}{5}}}$$

$z_k = -i\cot\frac{k\pi}{5}$ $\quad k = -2, -1, 1, 2$

Roots are $\pm i\cot\pi/5$, $\pm i\cot 2\pi/5$

7. $$\sum_{r=0}^{10} r\,{}^{10}C_r\, 3^r (-2)^{10-r}$$

$$= 10\sum_{r=1}^{10} {}^9C_{r-1}\, 3^r (-2)^{10-r} \;=\; 10.3\sum_{r=1}^{10} {}^9C_{r-1}\, 3^{r-1} (-2)^{10-r} \;=\; 30(3-2)^9 \;=\; 30$$

8. Given numbers can be rearranged as
1, 4, 7, ... (3n – 2)
2, 5, 8 , ... (3n –1)
3, 6, 9 ...... 3n
that means we must choose two numbers from last row or one number each from the first and second rows.
Therefore, the total number of ways is

$${}^nC_2 + {}^nC_1\,{}^nC_1 = \frac{n(n-1)}{2} + n^2 = \frac{3n^2-n}{2}$$

9. $\sin^2\alpha + \sin^2\beta + \sin^2\alpha$

$$\frac{\tan^2\alpha}{1+\tan^2\alpha} + \frac{\tan^2\beta}{1+\tan^2\beta} + \frac{\tan^2\gamma}{1+\tan^2\gamma}$$

$$\frac{x}{1+x} + \frac{y}{1+y} + \frac{z}{1+z} \quad (x = \tan^2\alpha,\ y = \tan^2\beta,\ z = \tan^2\gamma)$$

$$\frac{(x+y+z)+(xy+yz+zx+2xyz)+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)} \Rightarrow \frac{1+x+y+z+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)} = 1$$

10. (3)

$$|x| + |y| = 4\ ,\ \sin\left(\frac{\pi x^2}{3}\right) = 1 \Rightarrow |x|\ ,\ |y| \in [0, 4]\ ,\ \frac{\pi x^2}{3} = (4n+1)\frac{\pi}{2}$$

$$x^2 = \frac{3\pi}{2}(4n+1) = \frac{3\pi}{2} \text{ as } |x| \le 4 \Rightarrow |x| = \sqrt{\frac{3\pi}{2}}\ ,\ |y| = 4 - \sqrt{\frac{3\pi}{2}}$$

Thus these are 4 ordered pair

11. $3^{2n+2} - 8n - 9$

$= 9^{n+1} - 8n - 9$

$= (8+1)^{n+1} - 8n - 9$

$= {}^{n+1}C_0 8^{n+1} + {}^{n+1}C_1 8^7 + \text{-----} + {}^{n+1}C_{n-1} 8^2 + {}^{n+1}C_n\ 8 + {}^{n+1}C_{n+1} 8^0 - 8n - 9$

$= 1 + (n+1)\,8 + 8^2$ (integer) $- 8n - 9$

$= 8^2$ (Integer)

$\therefore n \in N$

12. P(1), P(2) and P (3) are not true

P(4) is true and $2^K < K!$

$\Rightarrow 2 \times 2^K < 2 \times K! \le (K+1)(K!)$ for $K \ge 1$

($\because 2 \le K + 1$ for $K \ge 1$)

13. $2x + 2y = 20 - z \Rightarrow x + y = 10 - \frac{z}{2}$

So coefficient of $x^{(10 - z/2)}$ in $(1 + x + x^2 + \ldots\ldots)^2$

$= x^{10-z/2}$ in $(1-x)^{-2} = 11 - \frac{z}{2}$

putting $z = 0, 2, 4, \ldots\ldots\ldots\ldots 20$

So total solutions $= 11 + 10 + 9 + \ldots\ldots + 1 = 66$

14. $$\frac{x^2}{\frac{27}{12}} - \frac{y^2}{\frac{27}{4}} = 1$$

$$e = \sqrt{1+\frac{12}{4}} = 2$$

$f \equiv (\pm 3, 0)$

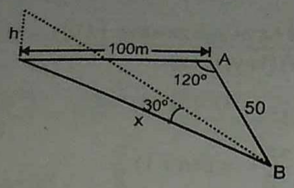
$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \quad e = \sqrt{1-\frac{16}{25}} = \frac{3}{5} \Rightarrow F = (\pm 3, 0)$$

The focii of two conic coincide. Hence two conic are confocal and therefore orthogonal

15. $\because\ z = -2 + 2\sqrt{3}\,i = 4\omega$

$\therefore\ z^{2n} + 2^{2n}.\,z^n + 2^{4n} = 4^{2n}.\,\omega^{2n} + 2^{2n}.\,4^n.\,\omega^n + 2^{4n}$

$$= 4^{2n}(\omega^{2n} + \omega^n + 1) = \begin{cases} 0 \text{ if } n \text{ is not a multiple of } 3 \\ 3.4^{2n} \text{ if } n \text{ is a multiple of } 3 \end{cases}$$

16. 

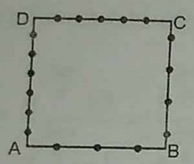
$$\therefore\ -\frac{1}{2} = \frac{100^2 + 50^2 - x^2}{2 \times 100 \times 50}$$

$x^2 = 17500 \quad \Rightarrow x = 50\sqrt{7}$

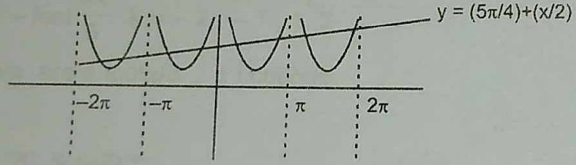
$\therefore\ \tan 30° = \frac{h}{x} \Rightarrow h = \frac{50\sqrt{7}}{\sqrt{3}}$ m

17. $x_i = 3,4,5,6,7$

$$A = \frac{\sum x_i}{n} = \frac{25}{5} = 5 \quad \Rightarrow \text{Mean deviation} = \frac{\sum |x_i - A|}{n} = \frac{2+1+0+1+2}{5} = \frac{6}{5} = 1.2$$

18. 

number of triangles = 3.4.5 + 4.5.6 + 5.6.3 + 3.4.6 = 342

19. 

Total 8 solutions

20. $C_1 \equiv (-g, -f)$, $r_1 = \sqrt{g^2 + f^2}$

13. f(x) is defined if $\log_{(x^2-1)} x \geq 0$, $x > 0$, $x^2 - 1 > 0$

and $x^2 - 1 \neq 1$

Now $\log_{(x^2-1)} x \geq 0$

$$\begin{cases} x \geq 1 & \text{if } x^2 - 1 > 1 \\ 0 < x < 1 & \text{if } x^2 - 1 < 1 \end{cases}$$

$$\begin{cases} x \geq 1 & \text{if } x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty) \\ 0 < x < 1 & \text{if } -\sqrt{2} < x < \sqrt{2} \end{cases}$$

so $x > \sqrt{2}$ or $0 < x < 1$

$x \in (0, 1) \cup (\sqrt{2}, \infty)$

and $x > 0$, $x^2 - 1 > 0$ and $x^2 - 1 \neq 1$

$x > 1$, $x \in (-\infty, -1) \cup (1, \infty)$ and $x \neq \sqrt{2}, -\sqrt{2}$)

$x \in (1, \sqrt{2}) \cup (\sqrt{2}, \infty)$ .... (ii)

from equation (i) and (ii) we get $x \in (\sqrt{2}, \infty)$

14. $f(x) = \cos(x + 2x + 3x + ... + nx) + i\sin(x + 2x + 3x + ... + nx)$

$$f(x) = \cos\frac{n(n+1)}{2}x + i\sin\frac{n(n+1)}{2}x \quad \Rightarrow \quad f'(x) = \frac{n(n+1)}{2}\left[-\sin\frac{n(n+1)}{2}x + i\cos\frac{n(n+1)}{2}x\right]$$

$$f''(x) = -\left[\frac{n(n+1)}{2}\right]^2\left[\cos\frac{n(n+1)x}{2} + i\sin\frac{n(n+1)x}{2}\right] \quad \Rightarrow \quad f''(x) = -\left[\frac{n(n+1)}{2}\right]^2 f(x)$$

$$f''(1) = -\left[\frac{n(n+1)}{2}\right]^2 f(1) \quad \text{Now } f(1) = 1 \quad \Rightarrow \quad f''(1) = -\left[\frac{n(n+1)}{2}\right]^2$$

15. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$1 \geq P(A) + P(B) - P(A \cap B) \geq \frac{3}{4}$$

$$\Rightarrow P(A) + P(B) - \frac{1}{8} \geq \frac{3}{4} \Rightarrow P(A) + P(B) \geq \frac{7}{8}$$

As the max value of $(A \cap B)$ is $\frac{3}{8}$ we get

$$1 \geq P(A) + P(B) - \frac{3}{8}$$

$$P(A) + P(B) \leq \frac{11}{8}$$

$$\frac{7}{8} \leq P(A) + P(B) \leq \frac{11}{8}$$

16. $1 + bc + qr = 0$ .... (i)
$1 + ac + pr = 0$ .... (ii)
$1 + ab + pq = 0$ .... (iii)
multiply (i) by ap (ii) by bq (iii) cr
$ap + abcp + apqr = 0$
$bq + abcq + bpqr = 0$
$cr + abcr + cpqr = 0$
since abc and pqr occour in all the three equations putting abc = x and pqr = y we get
$ap + px + by = 0$
$bq + qx + by = 0$ .... (iv)
$cr + rx + cy = 0$
system (iv) must have a common solution ($\therefore$ system is consistent)

$$\begin{vmatrix} ap & a & p \\ bq & b & q \\ cr & c & r \end{vmatrix} = 0$$

Statement - 2 is true

17. $xdy - ydx = \sqrt{x^2 - y^2}\,dx$

$$\Rightarrow \frac{xdy - ydx}{x^2} = \sqrt{1-\left(\frac{y}{x}\right)^2}\,\frac{dx}{x} \Rightarrow \frac{d\left(\frac{y}{x}\right)}{\sqrt{1-\left(\frac{y}{x}\right)^2}} = \frac{dx}{x} \Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ell n\, x + C$$

$x = 1, y = 0 \Rightarrow C = 0$
$\therefore$ Solution is $y = x\sin(\ell nx)$

18. $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

19. $\int_0^{\infty}\left[\frac{[x]+1}{e^x}\right]dx = \int_0^1\left[\frac{1}{e^x}\right]dx + \int_1^2\left[\frac{2}{e^x}\right]dx + \int_2^3\left[\frac{3}{e^x}\right] + \ldots\ldots$

$= 0 + 0 + 0 + \ldots\ldots$

$= 0$

20. For maximum point of discontinuity $x^2 - ax + 1 = 0$ must have two distinct roots.
असततता बिन्दुओं के अधिकतम होने के लिए $x^2 - ax + 1 = 0$ के दो भिन्न भिन्न मूल होने चाहिए।

$\Rightarrow \Delta > 0 \quad \Rightarrow a^2 - 4 > 0$

$\Rightarrow a \in (-\infty, -2) \cup (2, \infty)$

21. $x - ay - az = 0$

$bx - y + bz = 0$

$cx + cy - z = 0$

for non zero solution अशून्य हल के लिए

$$\begin{vmatrix} 1 & -a & -a \\ b & -1 & b \\ c & c & -1 \end{vmatrix} = 0$$

$\Rightarrow 1 - bc - ab - ac - 2abc = 0$

$\Rightarrow a(b+1)(c+1) + b(a+1)(c+1) + c(a+1)(b+1) = (a+1)(b+1)(c+1)$

$\Rightarrow \frac{a}{a+1} + \frac{b}{b+1} + \frac{c}{c+1} = 1$

22. $\lim_{x\to 0} \frac{\sin^b x}{x^{c-a}}$

$\because$ limit exists. so $b = c - a \Rightarrow a + b = c$

23. $\frac{dy}{dx} + y\tan x = x\tan x + 1$

$\text{I.F.} = e^{\int \tan x\, dx} = e^{\ell n \sec x} = \sec x$

$y\sec x = \int (x\tan x + 1)\sec x\, dx$

$= \int (x\sec x\tan x + \sec x)\, dx = x\sec x + C \Rightarrow y = x + C\cos x \quad \Rightarrow \quad \frac{dy}{dx} = 1 - C\sin x$

at y-axis

$\frac{dy}{dx} = 1$

Hence tangents will be parallel

24. $\int_0^x f(x)\,dx = (f(x))^3$

$f(x) = 3(f(x))^2 f'(x) \Rightarrow 3\int f(x)f'(x)\,dx = \int dx$

$\frac{3(f(x))^2}{2} = x + c \quad \Rightarrow \quad (f(x))^2 = \frac{2}{3}(x + c) \Rightarrow$ A parabola

25. $f'(x) = f(x) \Rightarrow f(x) = Ce^x$
$f(0) = 2 \Rightarrow C = 2 \Rightarrow f(x) = 2e^x$

So $I = \int_0^1 2e^x(x+2e^x)\,dx = 2\int_0^1 e^x x\,dx + 4\int_0^1 e^{2x}dx = 2(xe^x - e^x)\Big|_0^1 + 2e^{2x}\Big|_0^1 = 2 + e^2 - 2 = e^2$

26. since $8^n - 7n - 1 = (7+1)^n - 7n - 1$
$= 7^n + {}^nC_1 7^{n-1} + {}^nC_2 7^{n-2} + \ldots.. + {}^nC_{n-1} 7 + {}^nC_n - 7n - 1 = 49\,({}^nC_2 + {}^nC_3.7 + \ldots.. + {}^nC_n 7^{n-2})$,
$\therefore\ 8^n - 7n - 1$ is a multiple of 49 for all $n \in N$
hence x contains elements which are multiple of 49 & y contains all the multiples of 49, so $x \subseteq y$

27. $$\lim_{x\to 0} \frac{2k^2\left(\frac{e^x-1}{x}\right)^4}{\frac{\sin\left(\frac{x^2}{k^2}\right)}{\frac{x^2}{k^2}}\ \frac{\log\left(1+\frac{x^2}{2}\right)}{\frac{x^2}{2}}} = 8 \Rightarrow \quad k^2 = 4 \quad \Rightarrow k = \pm 2$$

28. For the given system to have a non-trivial solution, we must have

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0 \quad \Rightarrow \quad \begin{vmatrix} 1 & k & 3 \\ 0 & -2k & -11 \\ 0 & 3-2k & -10 \end{vmatrix} = 0$$

[Applying $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$]

$$\Rightarrow \quad 20k + 11(3-2k) = 0 \quad \Rightarrow \quad k = \frac{33}{2}$$

29. $y\frac{dy}{dx} = k \Rightarrow ydy = kdx$

$\frac{y}{2} = kx + c$

$\therefore$ option (2) is correct

30. $$\int 4\cos\left(x+\frac{\pi}{6}\right)\cos 2x \cos\left(\frac{5\pi}{6}+x\right) dx = 2\int\left(\cos(2x+\pi)+\cos\frac{2\pi}{3}\right)\cos 2x\,dx$$

$$= 2\int\left(-\cos 2x - \frac{1}{2}\right)\cos 2x\,dx = \int\left(-2\cos^2 2x - \cos 2x\right)dx$$

$$= -\int(1+\cos 4x + \cos 2x)\,dx = -x - \frac{\sin 4x}{4} - \frac{\sin 2x}{2} + c = -\left(x + \frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right) + c$$

SOLUTIONS TO FULL SYLLABUS TEST- 03

Answers

1.	(4)	2.	(3)	3.	(1)	4.	(3)	5.	(2)	6.	(2)	7.	(3)
8.	(2)	9.	(1)	10.	(4)	11.	(1)	12.	(4)	13.	(2)	14.	(4)
15.	(1)	16.	(2)	17.	(2)	18.	(2)	19.	(2)	20.	(4)	21.	(2)
22.	(2)	23.	(2)	24.	(2)	25.	(4)	26.	(1)	27.	(2)	28.	(4)
29.	(4)	30.	(1)										

SOLUTIONS

1. $(1, 2) \in R$ but $(2, 1) \notin R$ so R is not symmetric
$(3, 1)(1, 2) \in R$ but $(3, 2) \notin R$ so R is not transitive.
$\therefore$ R is reflexive.

2. Required area $= \int_0^1 (2x - 2x^2 - x\ell nx)\,dx = \frac{7}{12}$

3. $f(x) = (\sin^{-1}x)^2 + (\cos^{-1}x)^2 = (\sin^{-1}x + \cos^{-1}x)^2 - 2\sin^{-1}x\cos^{-1}x$

$\Rightarrow \frac{\pi^2}{4} - 2\cos^{-1}x\left(\frac{\pi}{2} - \cos^{-1}x\right) \Rightarrow 2(\cos^{-1}x)^2 - \pi\cos^{-1}x + \frac{\pi^2}{4}$

$\Rightarrow 2\left[(\cos^{-1}x)^2 - \frac{\pi}{2}(\cos^{-1}x)\right] + \frac{\pi^2}{4} \Rightarrow 2\left[\left(\cos^{-1}x - \frac{\pi}{4}\right)^2\right] + \frac{\pi^2}{4} - \frac{\pi^2}{8}$

$\Rightarrow 2\left[\left(\cos^{-1}x - \frac{\pi}{4}\right)^2\right] + \frac{\pi^2}{8} \Rightarrow$ now $\cos^{-1}x \in [0, \pi]$

$f(x)_{max.}$ when $\cos^{-1}x = \pi$ $\quad f(x)_{max.} = \frac{5\pi^2}{4}$

$f(x)_{min.}$ when $\cos^{-1}x = \pi/4$ $\quad f(x)_{min.} = \frac{\pi^2}{8}$

Range $\in \left[\frac{\pi^2}{8}, \frac{5\pi^2}{4}\right]$

4. We know that $|\text{adj}A| = |A|^{n-1}$. Hence statement 2 is false
Now,

$|\text{adj}(\text{adj}A)| = |\text{adj}A|^{n-1} = (|A|^{n-1})^{n-1} = |A|^{(n-1)^2}$

then $|\text{adj}(\text{adj}(\text{adj}A))| = |\text{adj}(\text{adj}A)|^{n-1} = |A^{(n-1)^2}|^{n-1} = |A|^{(n-1)^3}$

Statement 1 is true

5. $I = \int_0^1 \cot^{-1}(1 - x + x^2)\,dx = \int_0^1 \tan^{-1}\left(\frac{1}{1 - x + x^2}\right)dx$

$$\int_0^1 \tan^{-1}\left(\frac{x-(x-1)}{1+x(x-1)}\right)dx = \int_0^1 [\tan^{-1}x - \tan^{-1}(x-1)]dx$$

$$= \int_0^1 \tan^{-1}x\,dx - \int_0^1 \tan^{-1}(x-1)dx = \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}(1-x)dx$$

$$I = \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}[1-(1-x)]dx$$

$$I = 2\int_0^1 \tan^{-1}x\,dx = \lambda\int_0^1 \tan^{-1}x\,dx$$

$\lambda = 2$

6. RHL $x = 3+h = \lim_{h\to 0}([3+h-3]+[3-3-h]-(h+3))$

$= \lim_{h\to 0}([h]+[-h]-h-3) = 0-1-3 = -4$

LHL $x = 3-h = \lim_{h\to 0}([3-h-3]+[3-(3-h)]-(3-h))$

$= \lim_{h\to 0}([-h]+[h]-3+h) = -1-3 = -4$

7. $u = \dfrac{2}{x-2}$ $f(u) = \dfrac{1}{(u-6)(u-11)}$

f(u) is discontinuous at u = 6 and u = 11

$x = 2$, $x = \dfrac{7}{3}$, $x = \dfrac{24}{11}$

8. $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$

$$f'(x) = \frac{1}{3}\left[\frac{1}{(x+1)^{2/3}} - \frac{1}{(x-1)^{2/3}}\right]$$

clearly f'(x) does not exist at x = 1, – 1

now f'(x) = 0

$(x-1)^{2/3} = (x+1)^{2/3}$

x = 0

clearly f'(x) = 0 for any other value of $x \in [0, 1]$

The value of f(x) at x = 0 is 2. Hence the greatest value of f(x) is 2.

9. Taking O as the origin, let the position vectors of A, B, C be $\vec{a}$, $\vec{b}$, $\vec{c}$ respectively. Then position vector of G_1, G_2, G_3 be $\dfrac{\vec{b}+\vec{c}}{3}$, $\dfrac{\vec{c}+\vec{a}}{3}$, $\dfrac{\vec{a}+\vec{b}}{3}$ respectively

$V_1 = \dfrac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$

$V_2 = [\overrightarrow{OG_1}, \overrightarrow{OG_2}, \overrightarrow{OG_3}]$

$$V_2 = \left[\frac{\vec b+\vec c}{3}\ \frac{\vec c+\vec a}{3}\ \frac{\vec a+\vec b}{3}\right] = \frac{1}{27}[\vec b+\vec c\ \ \vec c+\vec a\ \ \vec a+\vec b]$$

$$V_2 = \frac{2}{27}[\vec a\ \vec b\ \vec c]$$

$$V_2 = \frac{2}{27}.6\,V_1 \Rightarrow 9V_2 = 4V_1$$

10. System of equations has non-trivial solutions

$$\therefore \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow a^3 + b^3 + c^3 - 3abc = 0$$

$\Rightarrow (a + b + c)(a\omega^2 + b\omega + c)(a\omega + b\omega^2 + c) = 0$

$\therefore$ either $a\omega^2 + b\omega + c = 0$ or $a\omega + b\omega^2 + c = 0$

Hence roots of the equation $at^2 + bt + c = 0$ are ω and ω^2 i.e. $at^2 + bt + c = 0$ has imaginary roots.

11. $\frac{a+1+2+0}{4} = 1 \Rightarrow a = 1$

$\frac{2+b+1+0}{4} = 2 \Rightarrow b = 5$

$\frac{3+2+c+0}{4} = -1 \Rightarrow c = -9$

$\Rightarrow P \equiv (1, 5, -9)$

Hence distance of P from origin is $\sqrt{107}$

12. The graph of P(x) is symmetric about y-axis at which it attains the minimum value. Hence rest of the extremums will occur in the multiples of two.

13. $$f'(x) = \frac{(\sin x+\cos x)(k\cos x-2\sin x)-(k\sin x+2\cos x)(\cos x-\sin x)}{(\sin x+\cos x)^2} = \frac{k-2}{(\sin x+\cos x)^2} > 0 \Rightarrow k > 2$$

14. Volume of the parallelopiped formed by $\vec a', \vec b', \vec c'$ is 4

$\therefore$ Volume of the parallelopiped formed by $\vec a, \vec b, \vec c$ is $\frac{1}{4}$

$$\vec b \times \vec c = \frac{(\vec c'\times\vec a')\times\vec c}{4} = \frac{1}{4}\vec a'$$

$$\therefore |\vec b \times \vec c| = \frac{\sqrt2}{4} = \frac{1}{2\sqrt2}$$

$$\therefore \text{length of altitude} = \frac{1}{4} \times 2\sqrt2 = \frac{1}{\sqrt2}.$$

15. $f'(x) = 7x^6 + 5x^4 + 6x^2 + 8$

$\forall x \in R,\ f'(x) > 0$

So f(x) is an increasing function.

$f(-\infty) = -\infty$ and $f(\infty) = \infty$ by mean value theorem.

It is clear that $f(x) = 0$ have at least one real root in between $(-\infty, \infty)$. But f(x) is increasing so there will be exactly one root in $x \in R$ for $f(x) = 0$

16. $f(x) = |x^2 - 13x + 40|$

$f(6) = |36 - 78 + 40| = |-2| = 2$

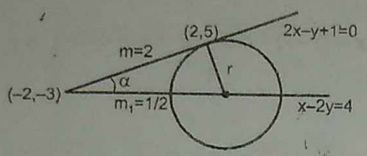
value of $x^2 - 13x + 40$ at $x = 6$ is -2

$\therefore$ near $x = 6$ $\quad f(x) = -x^2 + 13x - 40$

$\therefore$ $f'(x) = -2x + 13$ $\quad \therefore$ slope of the tangent = 1

$\therefore$ equation of tangent is $y - 2 = 1(x - 6)$ $\quad$ i.e $\quad y = x - 4$

17.



$\because \tan\alpha = \dfrac{2-\frac{1}{2}}{1+2\times\frac{1}{2}} = \dfrac{3}{4}$ $\quad \therefore \dfrac{r}{\sqrt{(4)^2+(8)^2}} = \dfrac{3}{4}$ $\quad \Rightarrow r = 3\sqrt{5}$

18. $\because \dfrac{dy}{dx} = \dfrac{-4x}{9y} = \dfrac{8}{9} \Rightarrow x = -2y \quad \therefore y = \pm\dfrac{1}{5}$

19. This is an isosceles Δ and therefore the median through C is the angle bisector of $\angle C$

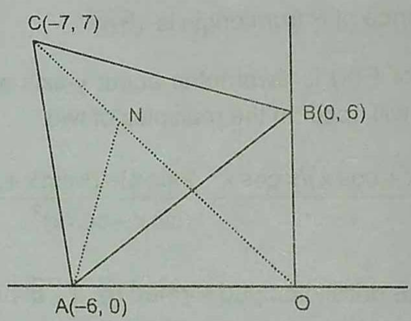
The equation of angle bisector is $y = -x$ and incentre $I = (-a, a)$ where $a > 0$

Equation of AC is $(y - 0) = -7(x + 6)$

$7x + y + 42 = 0$

Equation of AB is $x - y + 6 = 0$

Length of $\perp$ from I to AB and AC are equal



$$\left|\frac{-7a+a+42}{\sqrt{50}}\right| = \left|\frac{-a-a+6}{\sqrt{2}}\right|$$

$a = 9/2 \quad (a > 0)$

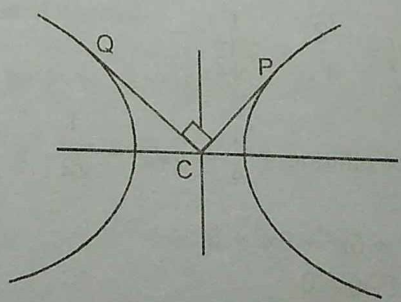
centre $\equiv \left(\dfrac{-9}{2}, \dfrac{9}{2}\right)$ and $r = \dfrac{3}{\sqrt{2}}$

Equation $\left(x+\dfrac{9}{2}\right)^2 + \left(y-\dfrac{9}{2}\right)^2 = \dfrac{9}{2}$

$x^2 + y^2 + 9x - 9y + 36 = 0$

20. $CP = r_1$ be inclined to TA at an angle θ so

P $(r_1\cos\theta, r_1\sin\theta)$ and P lies on hyperbola



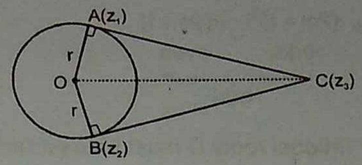
$$r_1^2\left(\frac{\cos^2\theta}{a^2} - \frac{\sin^2\theta}{b^2}\right) = 1$$

Replacing θ by $90 + \theta$

$$r_2^2\left(\frac{\sin^2\theta}{a^2} - \frac{\cos^2\theta}{b^2}\right) = 1$$

$$\frac{1}{r_1^2} + \frac{1}{r_2^2} = \cos^2\theta\left(\frac{1}{a^2} - \frac{1}{b^2}\right) + \sin^2\theta\left(\frac{1}{a^2} - \frac{1}{b^2}\right) = \frac{1}{a^2} - \frac{1}{b^2}$$

21. As ΔAOC is a triangle right angled at A



$$|z_3|^2 = |z_1|^2 + |z_3 - z_1|^2$$

$$2|z_1|^2 - z_1\bar{z}_3 - \bar{z}_1 z_3 = 0$$

$$2\,\bar{z}_1 - \bar{z}_3 - \frac{\bar{z}_1 z_3}{z_1} = 0 \quad \ldots\ldots\ldots\ldots (1)$$

similarly $2\bar{z}_2 - \bar{z}_3 - \frac{\bar{z}_2}{z_2} z_3 = 0 \quad \ldots\ldots\ldots (2)$

subtracting (2) – (1)

$$2(\bar{z}_2 - \bar{z}_1) = z_3\left(\frac{\bar{z}_1}{z_1} - \frac{\bar{z}_2}{z_2}\right) \Rightarrow \frac{2r^2[z_1 - z_2]}{z_1 z_2}$$

$$= z_3\, r^2\left(\frac{z_2^2 - z_1^2}{z_1^2\, z_2^2}\right) \Rightarrow z_3 = \frac{2z_1 z_2}{z_1 + z_2}$$

22. $b^2cx^2 - ab^2x + a^3 = 0$

multiply the given equation by $\frac{c}{a^3}$

$$\frac{b^2c^2}{a^3}x^2 - \frac{b^2c}{a^2}x + c = 0 \quad \Rightarrow \quad a\left(\frac{bc}{a^2}\right)^2 x^2 - b\left(\frac{bc}{a^2}\right)x + c = 0$$

$$\frac{bc}{a^2}x = \alpha\,,\,\beta \quad \Rightarrow \quad (\alpha + \beta)\,\alpha\beta\, x = \alpha,\, \beta$$

$$x = \frac{1}{\alpha(\alpha+\beta)}\,,\,\frac{1}{\beta(\alpha+\beta)} \quad \Rightarrow \quad \text{Roots are } \frac{1}{\alpha^2 + \alpha\beta}\,,\,\frac{1}{\beta^2 + \alpha\beta}$$

23. Given series ${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + {}^{20}C_3 + {}^{20}C_4 + {}^{20}C_5 + {}^{20}C_6 + {}^{20}C_7 + {}^{20}C_8$

$$= \frac{(2^{20} - {}^{20}C_{10})}{2} - {}^{20}C_9 = 2^{19} - \frac{({}^{20}C_{10} + 2.{}^{20}C_9)}{2}$$

24. 'P_1' must win atleast (n + 1) games let 'P_1' wins n + r games (r = 1, 2, ...n). Therefore, corresponding number of ways is ${}^{2n}C_{n+r}$. The total number of ways

$$\sum_{r=1}^{n} {}^{2n}C_{n+r} = {}^{2n}C_{n+1} + {}^{2n}C_{n+2} + \ldots\ldots + {}^{2n}C_{2n} = \frac{2^{2n}}{2} - {}^{2n}C_n = \frac{1}{2}\left(2^{2n} - 2\,{}^{2n}C_n\right)$$

25. $z = e^{i\theta} \Rightarrow \frac{z}{1 - z^2} = \frac{1}{\frac{1}{z} - z} = \frac{1}{-2i\sin\theta} = \frac{i}{2\sin\theta}$ lies on the y-axis

26. $D = \underbrace{\underset{\text{odd}}{(2m+1)^2} - \underset{\text{even}}{4(2n+1)}}_{\text{odd}}$

For rational roots D must be a perfect square

D is odd, let D be a perfect square of $(2\ell + 1)$, $\ell \in Z$

$(2m + 1)^2 - 4(2n + 1) = (2\ell + 1)^2$

$(2m + 1)^2 - (2\ell + 1)^2 = 4(2n + 1)$

$(2m + 1 + 2\ell + 1)(2m - 2\ell) = 4(2n + 1)$

$(m + \ell + 1)(m - \ell) = \underbrace{2n+1}_{\text{odd}}$

and LHS is always even and hence D cannot be perfect square. So roots cannot be rational

Hence, Statement - 1 is true, Statement - 2 is true and statement -2 is correct explanation of statement-1

27. $T_{r+1} = {}^{100}C_r \cdot \left(5^{\frac{1}{6}}\right)^{100-r} \cdot \left(2^{\frac{1}{8}}\right)^{r} = {}^{100}C_r \cdot 5^{\frac{100-r}{6}} \cdot 2^{\frac{r}{8}}$

For rational terms r = 16, 40, 64, 88

Number of rational terms = 4

$\therefore$ Number of irrational terms = 97

28. Since the line is a focal chord for the parabola hence tangents at the extremiters of it will intersect at right angle.

29. $p \Rightarrow (\sim p \vee q)$ is the false means p is true & $\sim p \vee q$ is false

$\Rightarrow$ p is true & both ~ p & q are false $\Rightarrow$ p is true & q is false

30. Mode is the measure of central tendency